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A new approach to numerical simulation of charge transport in double Gate-MOSFET☆

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ABSTRACT

We propose and describe in detail an effective numerical algorithm for finding the stationary solution of charge transport problem in a DG-MOSFET. Hydrodynamical models describing the process of charge transport in semiconductors are sets of nonlinear PDE's with small parameters and specific conditions on the boundary of transistor that essentially complicates the process of numerical simulations. We construct a new algorithm based on the stabilization method and ideas of approximation without saturation and pseudo-spectral methods that enables one to overcome all of the mentioned difficulties. The proposed algorithm enables us to obtain the solution for different geometrical characteristics of DG-MOSFET and boundary conditions (including the non-symmetric cases) with extremely small values of dimensionless doping density and dielectric constant that are used in practice.

1. Introduction

At the present time sizes of semiconductor electron devices become smaller and smaller up to micro and nano scales. At the same time, due to the complication of fabrication methods one needs to develop theoretical bases, new mathematical models and methods for designing such devices. Indeed, an adequate predictive modeling of processes taking place in a semiconductor requires more and more sophisticated mathematical models accounting for kinetic and quantum effects. Since the study of such models by analytical methods turns out to be technically impossible, methods of predictive numerical modeling are often used. However, applied problems dictate ever more rigorous values of parameters and complicated conditions on the boundaries of semiconductor devices. Many of today's computational algorithms turn out to be not efficient in such a situation. The aim of our work is just the construction of a new algorithm for solving such kind of difficult numerical problems.

Here we consider a class of hydrodynamical models (some of such models are described in [1–4]). We shall focus on a hydrodynamical MEP model describing with high rate of reliability the process of charge transport in semiconductors proposed in ([5,6]). This model is a set of quasilinear non-stationary PDE's written in the form of conservation laws obtained from the system of moments of the Boltzmann transport equation. While constructing this model the Maximum Entropy Principle (or MEP) was used for closing the system of moments. MEP model has been formulated in [5,6] for

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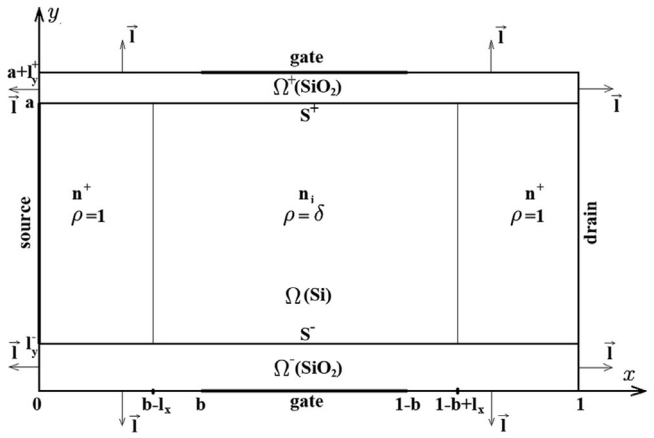

Fig. 1. Schematic sketch of silicon 2D DG-MOSFET.

the general Kane dispersion relation with cumbersome constitutive integral equations for the fluxes that leads to complex non-explicit expressions of the resolving PDEs of the model. In order to simplify the formulation and focus on designing of computational methods, following the ideas of paper [8], we restrict our self by using the limit case of parabolic band only. In this case, the equations of 2D hydrodynamical MEP model in a dimensionless form are the following:

$$\left.\begin{array}{l} R_t + \mathrm{div}(\mathbf{J}) = 0, \\ \mathbf{J}_t + \frac{2}{3}\nabla(RE) = R\mathbf{Q} + c_{11}\mathbf{J} + c_{12}\mathbf{I}, \\ (RE)_t + \mathrm{div}(\mathbf{I}) = (\mathbf{J}, \mathbf{Q}) + cR\sigma, \\ \mathbf{I}_t + \nabla(\frac{10}{9}RE^2) = \frac{5}{3}RE\mathbf{Q} + c_{21}\mathbf{J} + c_{22}\mathbf{I}. \end{array}\right\} \tag{1}$$

Here R is the electron density, E is the electron energy, $\mathbf{J} = R\mathbf{u}$, $\mathbf{I} = R\mathbf{q}$, $\mathrm{u} = (u^{(x)}, u^{(y)})$ is the vector of electron velocity in the Cartesian coordinate system (x, y), $\mathbf{q} = (q^{(x)}, q^{(y)})$ is the energy flux, $\sigma = 2E/3 - 1$, $\mathbf{Q} = \nabla\varphi = (\varphi_x, \varphi_y)$, $\varphi = \varphi(t, x, y)$ is the electric potential satisfying the equation

$$\Delta\varphi = \varphi_{xx} + \varphi_{yy} = \beta(R - \rho); \tag{2}$$

$\rho = \rho(x, y)$ is the doping density (a given function in the interior of the semiconductor). The coefficients c_{11},..., c_{22},c of system (1) are smooth functions of the energy E detailed in [7,8], $\beta > 0$ is an inverse dimensionless dielectric constant. We should emphasis here for comprehension that $\beta > 0$ is not the (inverse) static absolute permittivity equal to the permittivity of vacuum divided by permittivity of material. The expression for β was obtained after writing the equations of hydrodynamical MEP model in the dimensionless form (see (8)). It is worth noting that in certain domains of semiconductor devices the values of the constant β can be larger than other values of the problem by a factor of 10^6, the function ρ can take the values less than other values of the problem by a factor of 10^5–10^{10} (see (8), (9)). Note that for finding the approximate solutions of the hydrodynamical models a variety of numerical algorithms was proposed in [9–13]. The model was studied both analytically and numerically in [7,14–18].

This work deals with nonlinear problem of finding the stationary solution of charge transport problem in Double Gate-MOSFET (nano-size Double Gate-Metal Oxide Semiconductor Field Effect Transistor). Due to presence of small and large parameters, this problem is of high computational complexity. For solving it, we develop the method based on the specific combination of Chebyshev approximations and spline collocations. It is worth noting, that Chebyshev spectral methods have been developed and investigated both theoretically and numerically [19–21]. It has been discovered that the asymptotic of the truncation error of such methods strictly corresponds to those of the best polynomial approximations. This enables one to take into account an a priori property of smoothness of the desired solution that leads to significant increase in the rate of convergence and to minimization of errors even for problems with extremely steep gradients of the solution. In Russian works inspired by K. I. Babenko [22,23] this phenomenon is known as "*the absence of saturation of method*" or "*the method without saturation*" . In this work, for accurate and fast numerical simulation we propose an efficient combination of Chebyshev approximations without saturation, spline collocations and iterative stabilization (relaxation) technique.

2. Problem statement

We now formulate for Eqs. (1) and (2) the problem of finding the electric potential in a two dimensional DG-MOSFET. The detailed description of such a semiconductor transistor is given in [24]. A special feature of this transistor is the presence of two layers fabricated of silicon oxide. The DG-MOSFET in dimensionless variables x and y is sketched in Fig. 1.