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Pattern transitions in a vegetation system with cross-diffusion

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ABSTRACT

Regular pattern is a typical feature of vegetation distribution which can be recognized as early warnings of desertification. In this work, a vegetation system with cross diffusion is presented based on reaction-diffusion equations. By means of mathematical analysis, we obtain the appropriate parameter space which can ensure the emergence of stationary patterns. Moreover, it is unveiled that cross diffusion not only induces the pattern transitions, yet promotes the density of the vegetation. These obtained results suggest that cross diffusion is an important mechanism in vegetation dynamics.

1. Introduction

Vegetation is ubiquitous in our society and plays an important role in many aspects of our life [1–4]. We can easily list some examples around us. Firstly, it is well-known that vegetation absorbs energy and synthesizes starch from the sun's light based on photosynthesis, which produces sufficient food for human beings and animals [5,6]. That is to say, vegetation is one of the basic required elements for our life, survival and propagation. Besides, plants regulate the carbon and oxygen balance of environmental air. The garden plants in urban green space absorb carbon dioxide in the air by photosynthesis, and then release oxygen into air in order to maintain the carbon and oxygen balance of urban air [7,8]. Again, many garden plants can also release germicidal substances, such as eugenol, turpentine and walnut quinone, so the content of bacteria in the air of green space is significantly lower than that of non green land [9]. In this sense, the benefit of the green space is of positive significance for the maintenance of clean and hygienic urban air. Just from these examples, it is clear that, from both theoretical and experimental points of view, the study of vegetation is of great significance [10–14].

Mathematical modeling becomes one of the most useful tools in exploring the mechanisms on vegetation pattern. Thus far, there have been lots of achievements focusing on pattern transition of vegetation. For example, Klausmeier used a two-variables model to show the regular and irregular patterns in semi-arid vegetation [15]. Yuval revealed that high water absorption and rapid diffusion of water in perennial herbs [16]. Of particular interest, this work showed that the phase transition between multi-steady states is not necessarily catastrophic and can be gradually phase-changed. Based on the observation data of mathematical model, the cause of fairy circles vegetation patch is explained as intraspecific competition and the scale dependent effect of vegetation between animals that capture from plants [17]. In spite of great progress of recent years, most of these existing literature mainly focuses on the local reactions or self-diffusion. Recently, it has been noticed in the literature that cross-diffusion is generally overlooked despite of its potential ecological reality and intrinsic

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theoretical interest [18–20]. Thus, in this paper, we will consider a spatial vegetation system combined with self- and cross-diffusion.

The paper is organized as follows. In Section 2, we obtain a spatial vegetation system, and interpret the biological meaning of these parameters of the model. In Section 3, we analyze the spatial model and derive the conditions for the emergence of regular patterns. In Section 4, by performing a series of simulations, we illustrate the emergence of Turing patterns. What is more, pattern transition between stationary patterns is obtained. Finally, we summarize the conclusion.

2. A spatial vegetation system

It is well-known that vegetation systems follow two principles: one is that vegetation dynamics can be decomposed into growth and death processes; the other one is the conservation of mass principle, representing that vegetation grows as a function of the water [15]. Based on these two rules, we have the following vegetation system with spatial effects:

$$\frac{\partial P}{\partial T} = RJWP^2 - MP + D_1\Delta P, \tag{1a}$$

$$\frac{\partial W}{\partial T} = A - LW - RWP^2 + D_2\Delta W, \tag{1b}$$

where P, W are vegetation and water density, respectively. Parameter A stands for the production rate of the water in natural environment, the term $-LW$ means water reduction at the rate L caused by evaporation of moisture and the term $-MP$ indicates that the plant may vanish through natural mortality with the mortality coefficient M. $\triangle = \partial^2/\partial X^2 + \partial^2/\partial Y^2$ is the usual Laplacian operator in two dimensional space and D_1, D_2 are vegetation and water diffusion coefficients. From biological point of view, all parameters are positive constants. More details can be found in [15]. It is revealed that the slope of hill is not the essential condition for vegetation pattern formation. Thus, we do not take account for advection of the water.

In nature environment, the tendency of the water would get closer to the plant due to its absorption by the root of plants, and hence the chase velocity of water may be proportional to the dispersive velocity of the plants. This phenomenon is called as cross-diffusion [18,19,21]. Moreover, the cross-diffusion coefficient is negative which means that water tends to diffuse along the direction of higher concentration of plant biomass. In this case, we have the following vegetation system with cross diffusion:

$$\frac{\partial P}{\partial T} = RJWP^2 - MP + D_1\Delta P, \tag{2a}$$

$$\frac{\partial W}{\partial T} = A - LW - RWP^2 + D_2\Delta(W - \beta_1 P). \tag{2b}$$

By taking the following transformation:

$$w = \frac{\sqrt{RJ}}{\sqrt{L}}W, \quad p = \frac{\sqrt{R}}{\sqrt{L}}P, \quad a = \frac{\sqrt{RJ}}{L\sqrt{L}}A, \quad t = LT, \quad \beta = \beta_1 J,$$
$$x = \frac{\sqrt{L}}{\sqrt{D_1}}X, \quad y = \frac{\sqrt{L}}{\sqrt{D_1}}Y, \quad m = \frac{M}{L}, \quad v = \frac{1}{\sqrt{D_1 L}}V, \quad \sigma = \frac{D_2}{D_1},$$

we derive the following equations containing dimensionless quantities:

$$\frac{\partial p}{\partial t} = wp^2 - mp + \Delta p, \tag{3a}$$

$$\frac{\partial w}{\partial t} = a - w - wp^2 + \sigma\Delta(w - \beta p). \tag{3b}$$

3. Mathematical analysis

By means of direct calculations, we find that system (3) has three equilibrium points:

(i) $E_0 = (0, a)$, which corresponds to the disappearance of plant populations;

(ii) $E_* = (p_*, w_*) = (\frac{2m}{a+\sqrt{a^2-4m^2}}, \frac{a+\sqrt{a^2-4m^2}}{2})$, which corresponds to co-existence of water and plant populations. And it is a unstable equilibrium point;

(iii) $E^* = (p^*, w^*) = (\frac{2m}{a-\sqrt{a^2-4m^2}}, \frac{a-\sqrt{a^2-4m^2}}{2})$, which corresponds to existence of water and plant populations.