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Equal-collision-probability-curve method for safe spacecraft close-range proximity maneuvers

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Abstract

An equal-collision-probability-curve (ECPC) method is developed in this paper to address the problem of safe spacecraft proximity maneuvers. Considering the uncertainties' influence, the ECPC, which represents the curve of equal-collision-probability-points in the space around the target spacecraft, is firstly established. It is optimal to maneuver along the gradient direction of the ECPC, which is the fastest change in the ECPC. To calculate this direction, a novel auxiliary function, which has the same gradient direction as the collision probability function, is proposed. Compared to traditional collision probability functions, the proposed function does not contain transcendental elements and hence the computational burden can be greatly decreased while maintaining the necessary accuracy. Then, the safe close-range proximity maneuver generated by ECPC method can be implemented along the estimated gradient direction. Analytical validation is performed to assess the use of such collision avoidance scheme for safety critical operations. Furthermore, an improved Linear Quadratic Regulator (LQR) is designed to track the reference trajectory and a Lyapunov-based analysis verifies the stability of the overall closed-loop system. Numerical simulations show that the novel ECPC method is more computationally efficient than traditional methods while maintaining the same accuracy. Moreover, the novel scheme can be easily validated to guarantee the safety of the mission.

Keywords: Collision probability; Close-range proximity; Collision avoidance; Nonlinear relative motions

1. Introduction

On-orbit servicing (OOS) has attracted much attention in recent years due to the increasing number of on-orbit failures (Flores-Abad et al., 2014; Wu et al., 2018). Aiming at extending the operational lifetime or enhancing the capabilities of space assets, OOS comprises on-orbit assembly, inspection, maintenance, etc. (Bevilacqua et al., 2011; Spencer et al., 2016; Huang et al., 2016). Close-range operations are not only an essential element of OOS, but also a key technology of distributed space missions (Barnhart et al., 2007, 2009; Dutta et al., 2012). In the close proximity phases, the relative distance between the two spacecraft is small, and the orbital planes of them are well aligned. Any deviation of the reference trajectory from the chaser spacecraft to the target spacecraft may lead to a collision. Thus, the associated stringent safety requirements are one of the most critical aspects for the close range proximity operations. To guarantee the safe operations, the artificial potential function (APF) is often utilized (Bevilacqua et al., 2011; Nag and Summerer, 2013; Palacios et al., 2015; Spencer et al., 2016; Ni et al., 2017; Huang et al., 2017;

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Nomenclature

O_{I}	the center of the Earth
X_{I}, Y_{I}, Z_{I}	the three axis in inertial coordinate
o	the target spacecraft's center of mass
x, y, z	the three axis in LVLH coordinate
$\boldsymbol{r}_{\mathbf{Chaser}}$	the position vector of chaser spacecraft in inertial coordinate
$\boldsymbol{r}_{\mathbf{Target}}$	the position vector of target spacecraft in inertial coordinate
t, t_1	the time parameter
x_t, y_t	the relative position parameter
v_{xt}, v_{yt}	the relative velocity parameter
u_x, u_y	the relative control parameter
$\boldsymbol{r}_{1-t}$	the relative position vector at time t
$\boldsymbol{v}_{1-t}$	the relative velocity vector at time t
$\boldsymbol{u}_{1-t}$	the relative control vector at time t
μ	the gravitational constant of the Earth
a	the semi-major axis of the orbit of target spacecraft
n	the angle velocity of target spacecraft
$\boldsymbol{X}$	the state vector
$\boldsymbol{X}_0$	the initial state vector
$\boldsymbol{A}$	the state dynamics matrix
$\boldsymbol{B}$	the control mapping matrix
$\boldsymbol{\Phi}(t, t_0)$	the transition matrix from
$\boldsymbol{\Phi}_{\boldsymbol{rr}}(t,t_0), \boldsymbol{\Phi}_{\boldsymbol{rv}}(t,t_0), \boldsymbol{\Phi}_{\boldsymbol{vr}}(t,t_0), \boldsymbol{\Phi}_{\boldsymbol{vv}}(t,t_0), \boldsymbol{\Phi}_{\boldsymbol{v}}(t,t_i)$	the component of the transition matrix
t_0	the initial time
t_f	the final time
$\bar{\boldsymbol{X}}$	the mean value of $\boldsymbol{X}$
$\delta\boldsymbol{X}$	the error of state vector
$\delta\bar{\boldsymbol{X}}$	the mean value of $\delta\boldsymbol{X}$
$\boldsymbol{C}_{\delta X_0}$	the covariance matrices of the initial navigation uncertainties in LVLH frame
$\boldsymbol{C}_{\delta v_i}$	the covariance matrices of the control uncertainties in LVLH frame
$\boldsymbol{C}_{\delta X}$	the uncertainty covariance matrix of the state vector
$\boldsymbol{C}_{\delta r_{1-t}}$	the uncertainty covariance matrix of the relative position vector
N	the number of control impulses
σ_{xt}, σ_{yt}	the uncertainty covariance matrix parameter of the relative position
$\sigma_{vxt}, \sigma_{vyt}$	the uncertainty covariance matrix parameter of the relative velocity
x_{2-t}, y_{2-t}	coordinates used in probability density function
Ω	the geometry area of the target
δt	the error of the time
$\delta\boldsymbol{r}_{1-t}$	the required deflections
R_0	the radius of the target's volume
$\boldsymbol{r}_{2-t}$	the relative position in area Ω
P_c	the value of two-dimension probability density function
∇	the gradient symbol
f, g, V_0, V_1, V_2, h	the auxiliary function
$K, K_1, K_3, d_0, M_1, M_2, \lambda_1$	the auxiliary parameters
$\boldsymbol{K}_3, \boldsymbol{K}_4, \boldsymbol{K}_6, \boldsymbol{S}_1, \boldsymbol{S}_2$	the auxiliary matrices
$\boldsymbol{Q}, \boldsymbol{R}$	semi-positive definite symmetric matrics
$\boldsymbol{K}_2$	the optimal feedback matrix in LQR controller
$\boldsymbol{K}_5$	the optimal feedback matrix in improved LQR controller
$\boldsymbol{G}_1$	the true gradient direction of ECPC
$\mathrm{norm}(^*)$	the mold of vector*
$\boldsymbol{G}$	the gradient of ECPC
$\boldsymbol{G}_0$	the estimated gradient direction of ECPC
θ_1	the angle of true gradient direction
θ_2	the angle of estimated gradient direction by ECPC method
θ_3	the angle of estimated gradient direction by APF method
λ_0	a positive constant that shapes the magnitude of repulsive potential
D_0	the radius of the hazardous zone
v_t^{paral}	the magnitude of relative parallel velocity
$\boldsymbol{n}_{t_1}^{paral}$	the unit vector of relative parallel position
D_s	the braking distance
$a_{\max}$	the maximum acceleration of the chaser spacecraft
$v_{t_1}^{perpen}$	the magnitude of the relative perpendicular velocity
$\boldsymbol{n}_{t_1}^{perpen}$	the unit vector perpendicular to $\boldsymbol{n}_{t_1}^{paral}$
$\boldsymbol{F}_{repel}$	the repel force
$\boldsymbol{F}_{oparal}$	the parallel force
$\boldsymbol{F}_{operpen}$	the perpendicular force
$\boldsymbol{v}_{1-t_0}^{paral}$	the relative parallel velocity when the spacecraft arrived at the boundary of the hazardous zone
$\boldsymbol{v}_{1-t_1}^{paral}$	the relative parallel velocity when the spacecraft flies in the hazardous zone
$\boldsymbol{v}_{1-t_f}^{paral}$	the relative parallel velocity when the spacecraft reaches the minimum relative distance
x_2	one zero point respect to $h(r_{1-t_1})$
x_3	the maximum value of the r_{1-t_1}
ζ_0	the minimum relative distance between the chaser spacecraft and the target spacecraft
$\boldsymbol{X}_a$	the actual relative state vector
$\boldsymbol{X}_p$	the reference relative state vector
e	the error between the actual relative state vector and the reference relative state vector
$\boldsymbol{u}_{\mathrm{T}}$	the control impulse for orbital transformation
$\boldsymbol{u}^*$	the optimal control in LQR controller
$\boldsymbol{u}_1^*$	the optimal control in improved LQR controller
K_{upper}	the upper bound on the $\boldsymbol{K}_6^{\mathrm{T}}\boldsymbol{K}_6$
m	the mass of the chaser spacecraft
J_0, J_1	the cost function
$\boldsymbol{X}_{a-1}(t)$	the "virtual" actual state vector
$\boldsymbol{U}$, ς, ψ	the auxiliary parameter in APF method

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