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Original articles

Recent progress in impulsive control systems☆

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Abstract

This paper overviews the research investigations pertaining to impulsive control systems (ICSs). We focus on the fundamental results and recent progress of ICSs. After reviewing the relative literature, this paper will provide a comprehensive and intuitive overview of ICSs. Six aspects of ICSs are surveyed including basic theory, Lyapunov stability, delayed ICSs, input-to-state stability (ISS), finite-time control, and state-dependent impulses. Based on this, the paper provides a reference for further research on ICSs.

Keywords: Impulsive control systems (ICSs); Delayed impulses; Input-to-state stability (ISS); Stability; Finite-time control

1. Introduction

In the past few decades, impulsive control in dynamical systems has received considerable attention and it has been widely used in many fields such as orbital transfer of satellite, dosage supply in pharmacokinetics, ecosystems management, synchronization in chaotic secure communication systems, see [73,88,107]. In many cases, a real system may encounter some abrupt changes at certain time moments and cannot be considered continuously. This abrupt changes is called the impulsive phenomenon, and it has been extensively investigated based on impulsive differential equations in the past several years [8,40,59,65,95,99]. Actually, the study of ICSs can be traced back to the beginning of modern control theory. Many impulsive control methods were successfully developed under the framework of optimal control and were occasionally called impulsive control. Its necessity and importance lie in that, the main idea of impulsive control is to change the states instantaneously at certain instants. Therefore, impulsive control can reduce control cost and the amount of transmitted information drastically. In addition, in many cases impulsive control can give an efficient way to deal with systems which cannot endure continuous control inputs. Or, in some

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applications it is impossible to provide continuous control inputs. For instance, sometimes even only impulsive control can be used for control purpose, a government cannot change savings rates of its central bank everyday; A deep-space spacecraft cannot leave its engine on continuously if it has only limited fuel supply. Moreover, impulsive control is more effective than continuous control in some cases, suppose that the population of a kind of bacterium and the density of a bactericide are two state variables of a system. We can control the population by instantaneously changing the density of the bactericide without enhance the drug resistance of bacteria which may be caused by continuous control. Therefore, the research of ICS has important theoretical value and practical significance.

Moreover, it has also been shown that impulses have bilateral effects on stability of a system as well as the time delay. Differential systems with impulses and time delays which are also called impulsive delay differential systems admit more complex structure and dynamics, and have been studied for at least thirty years, see [41,72,75]. According to our work [38], from the point of view of impulsive effects, the study on stability of impulsive delay differential systems generally can be divided into two classes: (1) impulsive control problem (ICP) and (2) impulsive perturbation problem (IPP). In the case, where a given delay system without impulses has a certain stability property, such as uniform stability, asymptotical stability, or input-to-state stability, and it can remain the corresponding stability property under impulsive perturbations which go against the stability, it is regarded as IPP, which is actually a class of robustness problem. Much relative work on IPP have been reported see [20,66,97]. While in the case, where a given delay system without impulse does not have the stability property to be desired, but it may possess it via proper impulsive control, it is regarded as ICP. For example, if a delay system is originally unstable or stable but not attractive, but it becomes stable or asymptotic stability via the impulsive control, then it is an ICP. Until now, there are many valuable results for impulsive control system [44,52,76,77,87]. For example, a class of recurrent neural networks with discrete and continuously distributed delays was considered in [52], and it shows that network models may admit a periodic solution which is globally exponentially stable via proper impulsive control strategies even if it is originally unstable or divergent. In Ref. [77] some finite-time stability (FTS) criteria for the delayed nonlinear systems are derived via the hybrid controller (including impulsive control and sampled-data control). [44] considered the lag synchronization problem of chaotic delayed neural networks via impulsive control.

By analyzing existing related literature, this paper will provide a comprehensive and intuitive overview for the impulsive control systems, such as the basic theory of ICS, the stability of delayed ICS, the ISS and FTS of ICS, and state-dependent impulses. Based on the review, this paper provides a reference for further research on impulsive control theory. The rest of this paper is organized as follows. In Section 2, some notation and definitions of different kinds of ICSs are presented. Section 3 covers the stability of ICSs and describes impulsive control and impulsive perturbation. Section 4 discusses the research significance and some theoretical results of the delayed ICS. Section 5 considers the ISS of ICS. Section 6 introduces the finite-time control of impulsive control systems, where FTS includes finite-time to zero and finite-time boundedness. Section 7 covers the state-dependent impulses. Section 8 concludes the paper and discuss the future research direction on this topics.

2. Impulsive control systems (ICSs)

Notations. Let $\mathbb{R}$ denote the set of real numbers. $\mathbb{R}_+$ denotes the set of positive numbers. $\mathbb{Z}_+$ represents the set of positive integer. $\mathbb{N}$ denotes the set of nonnegative integer. $\mathbb{R}^n$ is the n-dimensional real spaces equipped with the Euclidean norm $|\bullet|$. $\mathbb{R}^{n\times m}$ denotes the $n \times m$-dimensional real spaces. $A > 0$ or $A < 0$ denotes that the matrix A is a symmetric and positive definite or negative definite matrix, respectively. The notation A^T and A^{-1} denote the transpose and the inverse of A, respectively. If A, B are symmetric matrices, $A > B$ means that $A - B$ is positive definite matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum eigenvalue and the minimum eigenvalue of matrix A, respectively. I denotes the identity matrix with appropriate dimensions. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k (1 \leq k \leq n)$, $C(J, S) = \{\varphi : J \to S \text{ is continuous}\}$ and $C^1(J, S) = \{\varphi : J \to S \text{ is continuously differentiable }\}$. $\Lambda = \{1, 2, \dots, n\}$. Notation $\star$ always denotes the symmetric block in a symmetric matrix. A function $\alpha : [0, \infty) \to [0, \infty)$ is of class $\mathcal{K}$ if α is continuous, strictly increasing, and $\alpha(0) = 0$. In addition, if α is unbounded, it is of class $\mathcal{K}_\infty$. A function $\beta : [0, \infty) \times [0, \infty) \to [0, \infty)$ is of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(r, t)$ decreases to 0 as $t \to \infty$ for each fixed $r \geq 0$. We use $\overline{\mathbb{R}}$ and $\overline{\mathbb{R}_+}$ to denote the extended, and extended nonnegative, real numbers, respectively. We also use $\overline{\mathcal{A}}$ to denote the closure of the set $\mathcal{A}$, respectively.

Impulsive control is a class of control methods, which is based on impulsive differential equations. Based on impulsive effects and impulsive instances, ICSs can be classified into different types. In the following, we shall review the definition of impulsive control and concepts of different types of ICSs.

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