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Evaluation and validation of equivalent properties of macro fibre composites for piezoelectric transducer modelling

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ABSTRACT

Piezoelectric transducers based on macro fibre composites (MFCs) are widely used for energy harvesting, actuation and sensing because of the high conformability, reliability and strong piezoelectric effect of MFCs. Analytical or numerical modelling of the heterogeneous MFC as a homogenous material with equivalent properties is usually required to predict the performance of the transducers. However, the equivalent properties reported in the literature are not suitable for this purpose. This work proposes an equal power-output method to numerically evaluate the equivalent properties of d_{31} type MFCs for piezoelectric transducer modelling. Taking energy harvesting application as a study case, it departs from the traditional method by applying electric assumptions that ensure the equal voltage, electric charge, and thus equal power output between the heterogeneous and homogeneous MFCs. The equivalent properties were characterised through the finite element (FE) analysis of the MFC's representative volume element (RVE), which is the minimum periodic unit in the MFC and takes account all the constitutes. The validity of these equivalent properties for energy harvesting transducer modelling was verified by FE modelling as well as experimental testing. The application of the equivalent properties for actuation and sensing transducer modelling was analysed and validated. FE modelling results showed that a homogeneous RVE with the equivalent properties accurately simulated the energy harvesting and actuation behaviours of the heterogeneous RVE. The simulated power output of MFC-based strain energy harvesters matched the mean experimental results with a mean error of 2.5%. When used for actuation, the MFC produced a free strain of 0.93 $\mu\varepsilon/V$, which is close to the manufacturer specification.

1. Introduction

Piezoelectric transducers for energy harvesting [1,2], actuation [3,4] and sensing [5] are well-established applications of piezoelectric materials. The most commonly used piezoelectric materials for these applications are piezoelectric ceramics, polymers and composites. Piezoelectric ceramics, mainly PZT, have excellent piezoelectric properties and thus high efficiency in energy transduction. However, their extremely brittle nature limits their conformability to curved surfaces and the stress/strain they can safely withstand without damage [6,7]. Piezoelectric polymers, mainly polyvinylidene difluoride (PVDF), can sustain much higher strain than ceramics due to their intrinsic flexibility [8,9], but the weak piezoelectric effect severely restricts the performance of PVDF in application. Piezoelectric composites comprise a piezoelectric fibrous phase embedded in an epoxy matrix phase. One of the most successful such composites is macro fibre composite (MFC), which was developed by NASA and is produced by Smart Materials Corp. It consists of rectangular PZT fibres embedded in an epoxy matrix and sandwiched between two electrode/epoxy layers. The whole structure is then encapsulated by two Kapton layers. It holds high strength, flexibility and reliability while still maintain a strong piezoelectric effect, thus attracting great interests for industrial applications and in the academic community [10–13].

Analytical or numerical analyses are usually required to predict and optimise the performance of piezoelectric transducers. For MFC-based transducers, considering the detailed structure of MFC will lead to enormous complexity of the analytical and numerical models. A practical approach is to envisage the heterogeneous MFC as a homogeneous piezoelectric material assigned with equivalent material properties. However, because of the complex structure of MFCs, the determination of the equivalent elastic and electro-mechanical properties of the MFC is challenging and the manufacturer has provided only limited information [14]. This challenge has attracted a great deal of effort in the research community [15–17].

Various homogenization techniques have been developed to determine the equivalent properties of the MFCs, such as analytical

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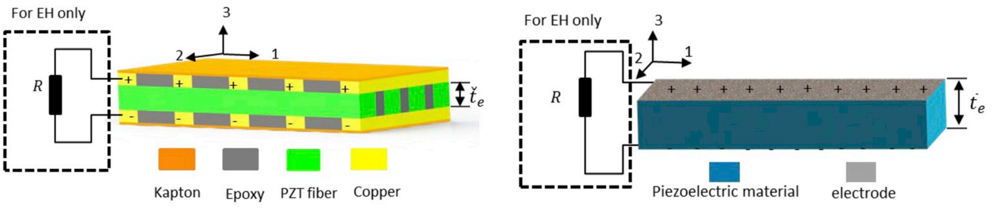

(a) (b)

Fig. 1. A schematic of (a) a real d_{31}-type MFC, which is heterogeneous and (b) the homogeneous MFC. The load resistors R in the dashed boxes are for energy harvesting applications.

mixing rules [15,18,19], asymptotic homogenization [20] and finite element (FE) homogenization. Deraemaeker et al. [15] proposed analytical mixing rules based on uniform field method to determine the equivalent properties of the active layer (PZT fibers embedded in epoxy matrix) of the MFC. Prasath et al. extended the application of the analytic mixing rules from the active layer [18] to the whole MFC including all the layers [19]. An asymptotic expansion homogenization method [20] was applied to compute the effective properties of d_{31} type MFC and showed comparable results with the analytical mixing rules. While the analytical methods become complex when considering all the layers of the MFC, FE analysis of MFC's representative volume element (RVE) [19,21–23] provides a good solution. It is applicable to general geometries and has been successfully applied to d_{31}, d_{33} and d_{15} type MFCs. A common feature of these homogenization methods is that they are all based on average quantities, i.e. the four field variables (electric field, electric displacement, stress and strain) of the homogeneous MFC are calculated as the volume-average of their counterparts in the heterogeneous MFC. They mainly differ in the techniques to determine these average quantities. For analytical mixing rules, the four field variables are assumed to be uniform within each constitute of the MFC. In case of an independent field variable, it is equal in each constitute and also equal to the corresponding average quantity. In case of a dependent field variable, the average quantity is a linear combination of the field variable in each constitute in terms of volume fraction. For FE homogenization, the field variables in a heterogeneous RVE subjected to specially designed boundary conditions are simulated and then averaged over the volume to derive the average quantities.

The aforementioned homogenization techniques result in relatively consistent equivalent properties and they have been successfully used to understand the global behaviours of MFCs [15–17] as well as their dependence on various parameters such as fibre volume fraction [19,24], bonding layer [25] and electric field strength [23]. However, as pointed out in Refs. [26,27], the equivalent piezoelectric and dielectric constants are dependent on the arbitrary chosen electrical assumption and boundary conditions. For these homogenization methods, the electric assumption is that the electric field and electric displacement of the homogeneous MFC is the volume-average quantities over the active layer [19,25] or the whole MFC [23,26,27]. The equivalent properties of the d_{31} MFC evaluated under this electric assumption may be difficult to use for transducer modelling, because the electric assumption has considered the equality or equivalent of the electric field and displacement, but not the equality of the voltage and electric charge between the heterogeneous and homogenous MFCs. For instance, in Refs. [23,26,27], the electric displacement of the homogeneous MFC was calculated by averaging the electric displacement produced solely by the PZT fibres over the whole MFC volume. Consequently, the electric displacement of the homogenous MFC, which is also the electric charge density, would be smaller than the heterogeneous. As a result, the homogenous MFC would have less electric charge than the heterogeneous. This could further lead to the inequality of other macroscopic parameters, e.g. voltage and electric power output for energy harvesting transducers. Steiger and P. Mokrý [28] have also found that when the MFC actuator is modelled as a homogenous piezoelectric material with identical geometrical dimensions, the d_{31} should be -267 pC/m. This value of d_{31} is significantly larger than the equivalent values ($\sim$-170 pC/m) reported in the literature using the aforementioned electric assumptions [19] [23].

This work proposes an equal power-output method for the first time to evaluate the equivalent properties of d_{31} type MFC for transducer modelling. It departs from the traditional method by applying electric assumptions that ensure the equal voltage, electric charge, and thus equal power output between the heterogeneous and homogeneous MFCs. Energy harvesting application is used as a study case. Finite element analysis of the RVE is performed to evaluate the equivalent properties, which are validated by FE modelling and experimental measurement of a strain energy harvester. The application of the equivalent properties for actuation and sensing is also analysed and validated. The methodology developed can also be used to evaluate the equivalent properties of other piezoelectric composites for transducer modelling.

2. Homogenization of MFCs: taking energy harvesting as a study case

A schematic of the d_{31} type MFC is shown in Fig. 1 (a). The MFC consists of 5 layers: an active layer with PZT fibres embedded in an epoxy matrix, two electrode layers with copper and epoxy phases, and two protective layers of Kapton. The PZT fibres are polled along 3-axis and metallized on surfaces normal to the 3-axis. Modelling the heterogeneous MFC directly is unwise because of the high computational cost due to the highly complex microstructure. An alternative way is to model the heterogeneous MFC as a homogeneous material, which has the same dimensions as the heterogeneous MFC and is assigned with equivalent macroscopic properties. The homogeneous MFC consists of a single layer of piezoelectric material polled along 3-axis, as shown in Fig. 1 (b).

The homogenization of the MFC is to evaluate the equivalent properties, with which the homogeneous MFC generates equal outputs of interest as the heterogeneous MFC when the same inputs are applied. The behaviours of the homogeneous MFC should comply with the constitutive equation of piezoelectric material, the stress-charge form of which is

$$\begin{Bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \\ \bar{D}_3 \end{Bmatrix} = \begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & \bar{c}_{13}^E & 0 & 0 & 0 & -\bar{e}_{31} \\ \bar{c}_{21}^E & \bar{c}_{22}^E & \bar{c}_{23}^E & 0 & 0 & 0 & -\bar{e}_{32} \\ \bar{c}_{31}^E & \bar{c}_{32}^E & \bar{c}_{33}^E & 0 & 0 & 0 & -\bar{e}_{31} \\ 0 & 0 & 0 & \bar{c}_{44}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}_{55}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{c}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{32} & \bar{e}_{33} & 0 & 0 & 0 & \bar{\varepsilon}_{33}^S \end{bmatrix} \begin{Bmatrix} \bar{S}_1 \\ \bar{S}_2 \\ \bar{S}_3 \\ \bar{S}_4 \\ \bar{S}_5 \\ \bar{S}_6 \\ \bar{E}_3 \end{Bmatrix} \tag{1}$$

where $\bar{c}_{ij}^E$ with i, $j = 1, \cdots, 6$ are the equivalent elastic constants at zero electric field ($\bar{E}_3 = 0$); $\bar{e}_{3p}$ with $p = 1,2,3$ are the equivalent piezoelectric stress constants; $\bar{\varepsilon}_{33}^S$ is the equivalent dielectric constant at blocked strain; $\bar{S}_j$, $\bar{T}_j$, $\bar{D}_3$ and $\bar{E}_3$ are the strain, stress, electric displacement and electric field in the homogenous MFC, respectively. It should be noted that the in-plane electric field is omitted ($\bar{E}_1 = \bar{E}_2 = 0$) in Eq. (1) because the MFC is polled along 3-axis. In this study, symbols with an accent of ‘¯’ and ‘ˇ’ respectively represent variables for homogeneous