Analytical and experimental study on deformation of thin-walled panel with non-ideal boundary conditions

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ABSTRACT

Based on the Kirchhoff-Love's assumptions, shell equations with non-ideal boundary conditions are established to describe the deformation of thin-walled aircraft panel resulting from positioning variation and clamping force in the assembly process. The shell equations are simplified by the theory of functions of a complex variable, and the analytical solutions are presented by the Fourier series approach and Galerkin method. Meanwhile, the potential functions are introduced to calculate the particular solution, which successfully avoids employing specialized displacement functions with different types of loads. A coordinate transformation model is also developed to transform the actual displacement and rotation boundary constraint in a Cartesian coordinate system into the non-ideal boundary in the curvilinear coordinate system. To verify the general applicability of the proposed method, analytical calculation, the finite element (FE) simulation and experiment in real application have been performed taking into account simply supported, ideal boundary and non-ideal boundary conditions, respectively. The results have shown the good performance of the accuracy of presented solution.

1. Introduction

Thin-walled structures as thin shells are widely used in engineering fields such as automotive, marine, chemical industry and other structural applications, especially in the aerospace engineering. Aircraft panels are the typical shell structures joined with the longitudinal and circumferential stiffeners by rivets or bolts to assemble the fuselage. Due to the intrinsic properties of low stiffness, panel skin is prone to elastic deformation or buckling in the positioning, clamping, drilling and joining operations. Actual outline of deformed panel varies from the ideal designed dimension, which seriously affects the aerodynamic performance and fatigue life of entire aircraft. At the fundamental assembly stage, the deformations of subcomponents are closely associated with different types of loads and boundary conditions. However, deriving the explicit deformation formula for a shell with non-ideal boundary conditions is a very challenging task for the prediction of dimensional variation occurring in panel assembly process which is essential for the final aircraft quality.

Numerous approaches have been presented by engineers and experts to tackle the deformation and variation analysis problem encountered in aircraft manufacturing. Saadat et al. [1] carried out experimental studies and finite element method (FEM) to evaluate overall deformation between the alignment of rib foot and skin during the part-to-part assembly. Liu et al. [2] employed the method of influence coefficient (MIC)

to analyze the effects of riveting process parameters on the sidewall panel key character (KC) deviation with a comparison of numerical and experimental methods. Corrado and Polini [3] analyzed the tolerance of tail beam components assembly using the Monto Carlo simulation method. Although the previous investigations applying FEM or statistical approaches could obtain the numerical results of structure deformation with the complicated boundary condition and combined load, the calculation accuracy relies on the element size or sample size. Yu et al. [4] obtained the normal vector of deformed panel by calculating the cross product of any two non-parallel vectors which were constructed by the four laser projection points on the panel surface. This method needed the large-volume data of geometrical points on the surface to achieve accurate calculation.

To solve this problem, the mechanical model of a flexible component was proposed to calculate the panel surface deformation effectively and rapidly. Su and Cesnik [5] implemented large deformation analysis of slender and flexible aircraft wings by introducing a nonlinear beam formulation, which was applicable to the structures with high slenderness ratio. For the panels with low slenderness ratios, different shell theories have been put forth to elucidate the intrinsic deformation mechanisms. Ivannikov et al. [6] presented a geometrically nonlinear thin shell model based on general 3D continuum mechanics principles and devised the boundary conditions of the special spatial shell with

Nomenclature

α, β, γ	axial, tangential, radial coordinates
x, y, z	the corresponding local orthogonal coordinates
u, v, w	displacement components of local coordinate system $Oxyz$
h, R	thickness of skin, curvature radius
θ	central angle of arc length β
θ_0	angle between axis z of local coordinate system at edge $\theta = 0$ and horizontal line
$\varepsilon_1, \varepsilon_2, \varepsilon_{12}$	strain components
χ_1, χ_2	the changes in curvatures
χ_{12}	the quantity represents the torsion of a differential element
E, μ, ρ	the modulus of elasticity, Poisson's ratio and density
N_1, N_2, N_{12}	normal forces and shear force
M_1, M_2, M_{12}	bending moments and twisting moment
q_1, q_2, q_3	external load components in the directions of x, y and z
D	a matrix of differential operators
$\|D_{ij}\|\ (i, j = 1, 2, 3)$	the cofactor of determinant $\|D\|$ in the ith row and the jth column
$L_{ij}\ (i, j = 1, 2, 3)$	linear differential operators
$\Phi_j\ (j = 1, 2, 3)$	the introduced potential functions respectively in three situations
$^{j}\Omega_1, {}^{j}\Omega_2, {}^{j}\Omega_3$	undetermined differential operators corrspond to situation j
∇^2	Laplace operator
u_0, v_0, w_0	displacements solutions of the corresponding homogeneous equations
$^{j}F(\alpha, \theta)\ (j = 1, 2, 3)$	the introduced potential functions for homogeneous equations in which the dependence of F on α, θ is separated
m	number of the terms remaining in the Fourier series expansion
$^{j}\Gamma_m(\alpha)$	the eigenfunctions of variable α
$^{j}\Xi_m(\theta)$	the eigenfunctions of variable θ
$^{j}\lambda_m$	the eigenvalue for differential operators
s	central angle of arc edge
$\varsigma_i\ (i = 1...8)$	solutions of characteristic equations of independent ODE
$^{m}J_i$	the real constants to be determined in function $^{j}\Gamma_m(\alpha)$
u_p, v_p, w_p	displacements solutions of the corresponding nonhomogeneous equations
$\Phi_{jp}\ (j = 1, 2, 3)$	the introduced potential functions for nonhomogeneous equations in which the dependence of Φ_{jp} on α, θ is separated
$\Phi_j^*(\alpha)\ (j = 1, 2, 3)$	the relevant function of variable α in the Fourier series expansions of Φ_{jp}
$q_{jm}(\alpha)\ (j = 1, 2, 3)$	the relevant function of variable α in the Fourier series expansions of q_j
$(\Delta x, \Delta y, \Delta z)$	displacement boundary constraints
$(\varphi_x, \varphi_y, \varphi_z)$	rotation boundary constraints
$\mathbf{P} = [\Delta x, \Delta y, \Delta z]^T$	translation vector
$\mathbf{\Psi}(\varphi_x, \varphi_y, \varphi_z)$	rotation matrix
ur, vr	rotation around axes α and β

lized the weighted residual methods to solve the governing equations in terms of displacement components of deformed thin shallow cylindrical panels. Semenov [9] expressed the total potential energy function and the equilibrium equations of geometrically nonlinear orthotropic shell. Chen et al. [10] determined the fundamental solution of partial differential equation (PDE) system modeling the elastostatic shell subjected to a concentrated point load. In most studies, shell deformations are described by highly coupled nonlinear PDEs which can be solved by the Ritz method [6,9,11], the Galerkin method [12–14], the partial fraction and Fourier transform techniques [10,15] and the extended Kantorovich method [16,17]. In the first two methods, displacement components are usually assumed in the form of the Fourier series which satisfy the boundary conditions. The assumed displacement potential functions are also introduced to uncouple and simplify partial differential equations of displacements. Then, these assumed solutions are substituted into the governing equations. Finally, a set of linear algebraic equations for many unknown Fourier coefficients or some ordinary differential equations are generated, which can be more easily solved [18]. Reddy [19] presented the Navier solutions, the assumed displacement admissible functions, to analyze the deflection and transverse shear stresses of rectangular plates with simply supported boundary. Chaudhuri et al. [20] derived the unknown integration constants of the stress resultants for an arbitrarily laminated cylindrical shell with admissible boundary conditions. The aforementioned literature review on the mechanical aspects of shells with various geometry features was mostly limited to harsh load distribution and simple boundary conditions (simply supported, free or clamped boundary condition), which are not feasible to the practical engineering application.

To predict the panel assembly deformation, a mechanical model of thin cylindrical shell subjected to combined loading and non-ideal boundary conditions is developed in the paper. Based on the Kirchhoff-Love's assumptions, explicit expressions for the displacement and rotation field of the deformed panel, namely the analytical solutions to the governing equations, are derived. The theory of functions of a complex variable is used to simplify the highly coupled nonlinear PDE. A methodology combined the generalized Fourier series approach and Galerkin method is proposed to address the boundary value problems for PDE. With the potential functions introduced, a more generalized procedure to calculate the particular solution of governing equation has been provided which can avoid searching specialized displacement functions for different load types. In addition, a coordinate transformation model is developed to convert the application boundary conditions to the non-ideal boundary in curvilinear coordinate system. Finally, the accuracy of presented solutions is verified by comparing with classical solutions, FEA results and the practical deformation measured in the experiment.

2. Model of thin-walled panels deformation with non-ideal boundary conditions

2.1. Problem formulation of aircraft panel assembly with positioning and clamping variation

The assembly system of fuselage panel is utilized to perform the positioning, clamping and measuring operations as shown in Fig. 1. The aircraft panel is composed of skin, stringers and frames which are assembled in the dedicated fixture. According to the theoretical position, the stringers are positioned and fixed by clamping mechanisms and the skin is fastened on the fixturing board by straining the puller straps. The assembly precision of the skin is significantly influenced by the position and rotation errors of the fixturing board. The laser tracker as measuring equipment is employed to capture the position data of points on the panel surface.

The entity panel can be split up into several elements by the contact edges of fixturing boards. The thickness of skin h is about 2 mm and the radius of curvature of the fuselage panel R is almost 2000 mm. So, the skin is regarded as a thin shell because the maximum value of the

the Kirchhoff-Love assumptions. The assumptions, suitable for isotropic material, contain that shell thickness is unchangeable and deflection is small with the normal to the middle surface constantly straight and perpendicular to mid-surface after deformation [7]. Alijani et al. [8] uti-