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Accuracy of vibro-acoustic computations using non-equidistant frequency spacing

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ABSTRACT

Within the vibro-acoustic optimisation of complex components under dynamic loading the radiated sound power is commonly used as an objective. For this purpose, the frequency-dependent sound power has to be quantified by a single scalar objective. For the required steady state simulations the mode-based frequency spacing can include non-equidistant step sizes as well as can change due to structural or material modifications. Thus, the total number of frequency steps is depending on the number of contributing modes that can be changed during optimisation processes with structural or material modifications. Furthermore, the accuracy of the objective has to be assured by choosing the required number of frequency steps and avoiding either under-resolved peaks or too many frequency steps.

In this study, we present an approach for the determination of the averaged sound power within the covered frequency range with non-equidistant spacing based on the power spectral density. These scalar quantities are robust to any model changes. Thereafter, the mean power is used as a convergence criterion to determine the number of required frequency steps for a single mode and thus to reduce the computational efforts to a minimum.

Further, a recommendation for a common rule for the spacing of single mode is given. This results in the frequency spacing estimation depending on the distance of neighbouring modes as well as the damping and biasing. Moreover, the combination of robust scalar objectives and efficient frequency spacing opens the prospects of accessing sound power objectives for complex optimisation problems.

1. Introduction

Lightweight components are typically thin-walled and stiff and thus tend to be sensitive for significant sound radiation. In addition, fibre-reinforced plastics (frp) with a wide range of adjustable material properties such as stiffness and even damping induced by manipulating layup, fibre/matrix material or fibre volume content are used.

Due to the acoustic sensitivity of stiff and thin-walled structures the sound radiation behaviour is a common optimisation objective within lightweight design [1–3]. Furthermore, the design of quiet structures implies a large potential of structural-acoustic optimisation. Fast frequency response analysis as well as its numerous repetitions with different parameter sets are a key issue in efficient optimisation processes [4,5].

Within these optimisation processes, the radiated sound power is used to express the radiation of components and machines and is formulated as the integral of the intensity over the radiating surface. Analytical solutions of sound power are limited to a few cases with regular geometries [6–8]. In addition, precise numerical approximation methods solving fluid–structure-interaction in one or both directions are used but are computationally expensive. Furthermore, the boundary element method (BEM) including fast-multipole techniques is a very popular approach for large-scale problems but is limited in applications with a large frequency range or modified structures within optimisation loops, e. g. [9].

In between fast analytical solutions of simple structures and computationally expensive BEM models are different approaches based on structural dynamic finite element analysis (FEA) using the surface velocity of the component. Particularly, there is the equivalent radiated power (ERP) as well as the more precise lumped parameter model (LPM) [10].

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These simplification methods are based on some of the following general assumptions. Stiff thin-walled structures with hard reflecting surfaces show identical particle velocity and normal structure velocity. Here, the sound pressure is evaluated on the structure's surface [10,11]. Last, the kinetic energy is implicitly given in steady state FEA solutions and thus provides information about the dynamic behaviour without any additional efforts. Possibilities and limits of estimating the radiated sound power by these methods have been shown by numerical studies on a composite component [12]. Moreover, the total power as an integral over frequency is used to demonstrate the feasibility and accuracy of such optimisation objectives.

As stiffness and damping are contradictorily influenced by fibre volume content, fibre orientation as well as stacking sequence optimising the material properties of frp results in non-linear dependencies [13,14]. In addition, composites show significant uncertainties in fibre orientation [15]. Thus, numerous finite-element simulations may be required either for genetic or gradient based optimisations. In summary, there is a need for efficient numerical quantities of structure-borne sound radiation of such processes [16].

Additionally, frequency domain structural dynamic FEA with modal superposition can be used to integrate anisotropic damping of composites by using an energy related approach [17–20] and is thus suitable for acoustic laminate optimisations. Moreover, the previous modal analysis can be used for a mode-based frequency spacing. This assures an accurate representation of the resonances but leads to non-equidistant frequency steps. The total number of frequency steps depends on the number of modes within the treated frequency range. Changing geometry or material parameters within optimisation procedures may result a varying number of contributing modes and thus different frequency stepping.

Reducing the radiated sound power by numerical optimisation methods requires a robust objective [4,21], typically as a scalar value. This reduction of information is in contrast to the strong dependency of the radiated sound power on the frequency wherein resonances contribute the most. Thus, a frequency-dependent assessment of the final design in relation to the excitation spectra is required.

Within this study, an approach to determine the average power in the given frequency range based on the power spectral density is presented. It is based on a subdivision of the sound intensity within the total bandwidth into small frequency intervals wherein only small changes of intensity appear [22]. The determination is independent of the number of modes and frequency steps and valid for varying frequency step sizes. This enables the comparison of different components, materials or geometries as well as the implementation as an optimisation objective.

Thereafter, the mean power is used as a convergence criterion to determine the number of required frequency steps for a single mode and thus to reduce the computational efforts to a minimum. As a result, the frequency spacing can be estimated depending on the distance of neighbouring modes as well as the damping and biasing.

2. Theoretical background of FEA-based sound power quantification

2.1. Structural FEA-based sound power approaches

The radiated sound power P is commonly used to quantify the structure-borne sound of vibrating parts. It is an integral of sound intensity I in normal direction over a closed surface Γ circumscribing the radiating object [10]

$$P = \int \vec{I} \cdot \vec{n} \, \mathrm{d}\Gamma \quad \text{with} \quad \vec{I} = \frac{1}{2}\Re(p\vec{v}^*) \tag{1}$$

wherein $*$ denotes a conjugate complex value as well as $\Re$ the real part of a complex state variable. The velocity normal to the surface $v_n = \vec{v} \cdot \vec{n}$ is determined by steady state structural dynamic FE-analysis. Hence, a simple, popular and efficient approach for the sound pressure in local relation is

$$\boldsymbol{p} \approx \rho_f \, c_f \, \boldsymbol{v}_n \tag{2}$$

with the fluid's density ρ_f as well as its speed of sound c_f. The relation between particle velocity and sound pressure is reduced to the fluid's characteristic impedance

$$Z_0 = \rho_f \, c_f. \tag{3}$$

The approximation by the equivalent radiated sound power (ERP) is typical in far fields and high frequencies and results in the sound power as a surface integral

$$P_{ERP} = \frac{1}{2}\rho_f \, c_f \int_\Gamma |\boldsymbol{v}_n|^2 \mathrm{d}\Gamma \tag{4}$$

which is eqivalent to a radiation efficiency of $\sigma = 1$ or a discretised formulation for N_e constant elements with an area S_μ

$$P_{ERP} = \frac{1}{2}\rho_f \, c_f \sum_{\mu=1}^{N_e} S_\mu v_{n_\mu} v_{n_\mu}^*. \tag{5}$$

This simple formulation is based on the assumption of the same radiation efficiency $\sigma = 1$ for all elemental sources. It neglects effects such as interaction between local sources. Generally overestimating the radiation, it gives a good impression of an upper bound for convex rigid bodies and high frequencies.

The most accurate approximation is the lumped parameter model (LPM) by Koopmann and Fahnline [23–25]. It is a simplification of the Rayleigh-integral including a Taylor series for the Green's function as a multi-pole expansion. This yields to a formulation for a source at x_μ and a receiver at y_ν

$$P_{LPM} = \frac{1}{2} k \, \rho_f \, c_f \sum_{\mu=1}^{N_e}\sum_{\nu=1}^{N_e} S_\mu S_\nu \frac{\sin(k|x_\mu - y_\nu|)}{2\pi|x_\mu - y_\nu|} \Re\{v_\mu v_\nu^*\} \tag{6}$$

also considering the interactions of the elemental sources. With the wave number k it includes a frequency-dependent radiation efficiency. The LPM predictions are exact for dipole modes. Besides, it already provides appropriate results in the low and mid frequency range.

In summary, the FEA-implementation of the sound power approximations is based on piecewise constant elements.

$$P = \sum_{\mu=1}^{N_e}\sum_{\nu=1}^{N_e} P_{\mu\nu} = \sum_{\mu=1}^{N_e} P_{\mu\mu} + 2\sum_{\mu=1}^{N_e-1}\sum_{\nu=\mu+1}^{N_e} P_{\mu\nu} \tag{7}$$

The sound power portions $P_{\mu\nu}$ are understood as a partial sound power of all N_e constant elements acting as a monopole source. In detail, $P_{\mu\mu}$ considers the independent source distributions whereas $P_{\mu\nu}(\mu \neq \nu)$ represents the interaction between these sources.

The matrix is symmetric and its elements can be determined by

$$P_{\mu\nu} = P_{\nu\mu} = \frac{1}{2}\rho_f \, c_f \, S_\mu \, \sigma_{\mu\nu} \, \Re\{v_\mu v_\nu^*\} \tag{8}$$

with the dimensionless radiation efficiency $\sigma_{\mu\nu}$. It gives a good impression of the different considerations of local effects and frequency-dependency. For the different sound power models $\sigma_{\mu\nu}$ acts as

$$\sigma_{\mu\nu} = \delta_{\mu\nu} \qquad \text{for ERP}, \tag{9}$$