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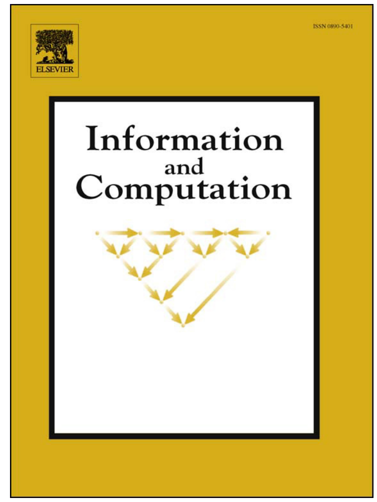


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# Inapproximability Results for Constrained Approximate Nash Equilibria☆

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**Abstract**

We study the problem of finding approximate Nash equilibria that satisfy certain conditions, such as providing good social welfare. In particular, we study the problem $\epsilon$-NE $\delta$-SW: find an $\epsilon$-approximate Nash equilibrium ($\epsilon$-NE) that is within $\delta$ of the best social welfare achievable by an $\epsilon$-NE. Our main result is that, if the exponential-time hypothesis (ETH) is true, then solving $\left(\frac{1}{8} - \mathrm{O}(\delta)\right)$-NE $\mathrm{O}(\delta)$-SW for an $n \times n$ bimatrix game requires $n^{\widetilde{\Omega}(\log n)}$ time. Building on this result, we show similar conditional running time lower bounds on a number of decision problems for approximate Nash equilibria that do not involve social welfare, including maximizing or minimizing a certain player's payoff, or finding approximate equilibria contained in a given pair of supports. We show quasi-polynomial lower bounds for these problems assuming that ETH holds, where these lower bounds apply to $\epsilon$-Nash equilibria for all $\epsilon < \frac{1}{8}$. The hardness of these other decision problems has so far only been studied in the context of exact equilibria.

*Keywords:* Approximate Nash equilibrium, constrained equilibrium, quasi-polynomial time, lower bound, Exponential Time Hypothesis.

## 1. Introduction

One of the most fundamental problems in game theory is to find a Nash equilibrium of a game. Often, we are not interested in finding any Nash equilibrium, but instead we want to find one that also satisfies certain constraints. For example, we may want to find a Nash equilibrium that provides high *social welfare*, which is the sum of the players' payoffs.

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