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# Towards an analytical formulation for fluid structure tank vibration analysis: Modal equivalency using granular materials

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ABSTRACT

This paper focuses on a new methodology for substitution of Liquid Hydrogen (LH2) contained in cryogenic tank in vibration analysis by using surrogate granular materials. Our analysis is limited to a 3D tank that is supposed to be fully filled with granular grains and tries to establish a modal equivalence between (tank fully filled with LH2) and (tank fully filled with granular grains) systems. For this, we determine required pre-stresses based on a homogenization technique from properties of grains. After reviewing some important mathematical formulations of vibration and homogenization model, an example of modal equivalence between these two systems is presented. Analytical results are also compared with numerical simulations in order to prove the suitability of the new method.

## 1. Introduction

Liquid Hydrogen (**LH2**) is largely used in cryogenic engines for launchers due to its high efficiency [1]. However, utilization of this fuel raises some critical technical problems linked not only to explosiveness and fugacity, but also to our understanding of the dynamic behaviour of the cryogenic tanks. LH2 is indeed too dangerous and expensive to be performed in vibration tests in laboratory [2]. While for other liquid fuels, surrogate liquids such as water are largely used to study the dynamic behavior of the tank, it is however difficult to apply the same strategy for LH2 due to its very low mass density.

A recent work performed by our research group [3] suggested the idea that pre-stressed grain systems as a surrogate material may possibly help to circumvent the aforementioned experimental issue. While outlining a new methodology for this, these preliminary studies provided indeed good analytical results for two-dimensional structural coupling cases, which tend to support the above hypothesis. Nevertheless, the potential applicability of this approach to more engineering cases of three-dimensional cylindrical cryogenic tanks still remained for investigation. That one constitutes the extension targeted by the present article.

Globally, the approach developed here, and which builds on the one used by Chiambaretto et al. [3], involves two main tasks aimed to define an *effective* strategy for the substitution of liquid fuel in vibration analysis by using granular materials, as schematized in Fig. 1. In one of the tasks, the vibration of *tank-fluid material* (**TFM**) system is considered as one of a *tank-solid material* (**TSM**) system while assuming that a modal equivalence between the vibrations of these two different structural/material systems can be found for some frequency bandwidths. In the second task, a modal equivalence between the vibrations of the TSM system and a *tank-granular material* (**TGM**) must be found with the help of a homogenization technique. The latter task requires notably determining the suitable pre-pressure that must be applied to an assembly of granular systems - to be modelled as a homogeneous continuum - that achieves the modal frequencies and mode shapes of a cylindrical tank filled with LH2. Besides, as performed before in [3], the aforementioned equivalences are stated by means of a simple and efficient methodology. The strategy and the solutions to the different considered dynamics problems that are notably obtained by Rayleigh-Ritz method rely on the five following hypotheses:

- **H1**: Acoustic effects and viscosity are negligible in the fluid domain (LH2). This assumption is notably based both on a Chiambaretto et al.'s estimate of the Helmholtz number [3] that was found to be $10^{-2}$, and on McCarty's one for the viscosity of LH2 at the storage state [21] that is equal to $120.10^{-7}$ kg/ms.
- **H2**: Pre-stressed pressure is high enough to prevent relative motions between grains.

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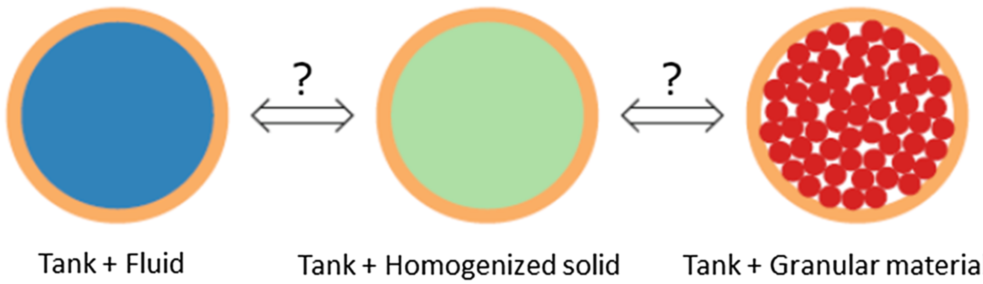


**Fig. 1.** Modal equivalence between fluid filled beam and grains filled beam using a homogeneous model for granular material [3]

- **H3**: Granular material can be described as an elastic material with material coefficient depending on pressure.
- **H4**: Pressure variation during vibration tests is low enough to linearize material coefficients.
- **H5**: The radial displacements of the mode shape of a free circular beam, of a circular membrane on its free edge and of a beam filled by this circular membrane are the same. The appropriateness of this assumption was checked numerically with COMSOL [3], the simulations pointing out that the modal shapes of a circular beam, of a membrane on free edge, and the coupled system, are very similar.

To our knowledge, the foregoing aim and methodology seem to be new in the literature on Tank – Granular material' coupling systems, where researches concentrate mainly on the storage and flow of grains in silos; the main goal of the proposed methodology is indeed to achieve modal equivalency (over some frequency bandwidths) between a *tank-fluid* material (TFM) system and a *tank-solid material* (TGM) one so that this last material system can be used as a surrogate material in vibratory tests for launcher tanks.

This paper contains three main sections. First, the general strategy, the mathematical formulations for the vibration analysis and the homogenization model are introduced in the 1st section. The 2nd section presents the analytical results for the vibration analyses of an empty tank, a tank fully filled with fluid or else with granular materials. Comparisons between analytical and numerical results are also provided to show the potential of the new methodology. Lastly, in the 3rd section, some important conclusions are drawn and proposals are made in order to find an effective strategy for a surrogate granular material in the future.

## 2. Mathematical formulations

### 2.1. Methodology for substitution of Liquid Hydrogen in vibration analysis

As already stated, the final purpose of the ongoing project is to find a surrogate granular material to substitute the liquid fuel in vibration tests, the concerned liquid fuel being LH2 in our targeted application case. The surrogate material must satisfy the condition that the most representative mode shapes and eigenfrequencies of tank filled with grains are the same as the ones of tank filled with liquid. For our study, the most representative mode shapes and eigenfrequencies of interest are related to the lowest frequencies of vibration, but the criterion of equivalence may be adjusted to other frequency ranges.

From a mathematical point of view, our criterion of equivalence is said to be fulfilled if we can find a granular material solving the following equation

$$\frac{K^T + K^G}{M^T + M^G} = \frac{K^T}{M^T + M^F} \tag{1}$$

with

- $K^T$, $M^T$: respectively the modal stiffness and the modal mass of the tank.
- $K^G$, $M^G$: respectively the modal stiffness, the modal mass of the granular assembly.
- $M^F$: modal mass added by the fluid.
- Based on this idea, the following scheme is proposed in order to find the required pre – stress value for each vibration mode.

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**Step 1**: Determine Modal shapes and Eigen frequencies of **TFM** system as targeted value for **TSM** system

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**Step 2**: Identify the parameters of an elastic homogenization model from an oedometric test. After this step, **TGM** system is considered as **TSM** system

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**Step 3**: Compute the mechanical properties such as Young's modulus and mass density of homogenized granular material during the compression

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**Step 4**: Determine modal mass and stiffness of granular assembly from mechanical properties found at step 3

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**Step 5**: Determine eigenfrequencies of **TGM** system by formula (1) and compare with targeted value given by **TFM** system. Once, a modal equivalency between the two systems is found, a/the required pre-stress is determined

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