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Thermal effect on vibration and buckling analysis of thin isotropic/orthotropic rectangular plates with crack defects

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ABSTRACT

The present research is concerned with the vibration analysis of thin isotropic and orthotropic rectangular plates with crack defects under thermal environmental conditions. In the literature, there are only few studies reported in this direction. Based on the classical plate theory, the governing equations of the isotropic and orthotropic cracked rectangular plates can be derived, in which a surface crack located at the plate center is formulated based on a line-spring model. Since the dynamic behavior of structural elements is significantly affected by thermal effects, a thermal buckling analysis for isotropic and orthotropic plates is also conducted. A uniform heating load on the cracked rectangular plates is considered and the critical buckling temperature of the plates with or without cracks is investigated. The discrete singular convolution (DSC) method is then applied to formulate the eigenvalue equations for the cracked rectangular plates under various thermal conditions. The DSC technique is an ingenious method in stability and dynamic analysis of plates, not only it is a flexible local method to handle complex geometries and boundary conditions, but also it performs as a global approach with a high degree of accuracy. To go beyond the limitation of the DSC method, the use of Taylor's series expansion method is incorporated for the treatment of free boundary conditions. In addition, this is the first attempt to explore its application on the analysis of cracked rectangular plates under thermal effects. In this work, the vibration of isotropic and orthotropic cracked rectangular plates with various combinations of boundary conditions is studied. A special restrained manner of simply supported conditions that are permissible for in-plane movements is also analyzed. The obtained solutions herein are compared with the existing results to verify the accuracy and reliability. Besides, accurate first-known solutions are also presented.

1. Introduction

Thin-walled rectangular plate-type structures are basic elements in structural and mechanical engineering. Although the dynamic and buckling behavior of thin plates have received great attention, many modern structures are made of composite materials due to their high strength-to-weight and stiffness-to-weight ratios. Composite materials can be generally modeled as orthotropic plates. The orthotropic properties are obtained by changing the isotropic characteristics along perpendicular directions and utilizing anisotropic materials during manufacturing processes. From a perspective view of mechanics, thin isotropic and orthotropic plates are often subject to various in-plane forces and moments by strong dynamic loads. A long-term influence from dynamic responses is to induce potential damages on such plates, resulting in the failure of structures. Therefore, the detection and localization of damages or cracks on thin isotropic and orthotropic plates are crucial for the safety requirement and good performance in their service life. Besides, the exposure of thin isotropic and orthotropic plates in thermal environments would affect the plate stiffness and change its dynamic behavior. Motivated by the need to address this issue, the understanding of the dynamic and stability analysis of cracked isotropic and orthotropic plates subject to thermal conditions is highly desired.

In the early scientific monographs [1–4], the dynamic characteristics of various structural elements have been intensively studied. The fundamental theories have been well established to provide useful guidelines for structural design and control. Stability and vibration studies for cracked thin plates have been developed in the past decades. Wang et al. [5] provided generic buckling solutions for Mindlin plates of various shapes, including polygonal, elliptical, semicircular and

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annular plates. For beam structures, Feng et al. [6] recently presented an investigation on the nonlinear free vibration of a multi-layer polymer nanocomposite beam reinforced by graphene platelets. On the other hand, many scholars have devoted to the investigation of crack effects on plates, involving part-through surface cracks, part-through finite length cracks, all part-through cracks and internal cracks [7–11]. For instance, Rice and Levy [7] introduced a line-spring model (LSM) based on the classical Kirchhoff plate theory to evaluate the effect of plate cracks. In their work, the part-through crack is represented by a continuously distributed line-spring. The appropriate compliance coefficients were chosen to match the tension and bending of an edge-cracked tip in terms of plane strains. Furthermore, Israr et al. [12] proposed an analytical model for isotropic rectangular plates with cracks using the LSM. According to the work from Aksel and Erdogan [13], they considered the stress intensity factor to account for internal cracks and established the crack compliance coefficients using a set of polynomial functions. Moreover, Joshi et al. [14] constructed an analytical model for the vibration analysis of internally cracked rectangular plates using the LSM, in which the compliance coefficients are variances with the ratio d/h (i.e., d is an offset distance between the crack center line and the neutral plane, and h is the plate thickness). Bose and Mohanty [15] extended the previous model [12] to investigate the dynamic characteristics of rectangular plates with an angled crack. They also took the effect of the far-field tensile stress on the crack tip bending stress into consideration.

In addition, there are research studies on the vibration of cracked plates by means of numerical techniques. Liew et al. [16] employed the domain decomposition method by utilizing appropriate shape functions for the analysis of cracked plates with typical boundary supports. Huang and Leissa [17] applied the Ritz method combined with various composition techniques to investigate the effects of crack location, crack length and orientation on the vibration of plates. Xu and Chen [18] used the differential quadrature finite element method to study the dynamic behavior of plates with irregular shaped cracks.

As aforementioned, composite material structures can be simplified as orthotropic plates. Many relevant research studies on orthotropic plates have been conducted. Xing and Liu [19] applied a separation of variables to obtain exact solutions for the free vibration of thin orthotropic plates. Biancolini et al. [20] investigated the approximate evaluation for the vibration frequencies of orthotropic plates. In their work, a particular form of the Rayleigh method was used to simplify the calculation procedures. Kurpa et al. [21] studied the free flexural vibration of thin orthotropic plates with mixed boundary conditions by the method of R-function. On the other hand, Joshi et al. [22] extended the analytical model developed by Israr et al. [12] to thin rectangular orthotropic plates with crack defects. They discussed the effects of crack length and boundary condition on the natural frequency of such plates. Yum and Hong [23] analyzed the mixed mode fracture problem for finite orthotropic plates with an inclined crack. Szekrényes [24] incorporated the classical laminated and first-order shear deformable plate theories to construct a double-plate system for the analysis of delaminated orthotropic composite plates. Shen et al. [25] utilized the Rayleigh-Ritz method to investigate the bending and vibration characteristics of a strengthened plate under various boundary conditions.

It is commonly known that engineering structures are usually exposed in thermal environment during their service life. The dynamic and stability behaviors of plate structures can be adversely affected by thermal conditions. In an early effort, Jadeja and Loo [26] investigated the thermally induced vibration of rectangular plates with one edge clamped and all other edges simply supported. They applied the Galerkin method to obtain the deflection curve of the plates. Shen [27] analyzed the thermal post-buckling behavior of imperfect shear deformation laminated plates. The thermal loads can be produced as a uniform temperature rises. More recently, Zhou et al. [28] used the Hamilton principle to derive the governing equation of motion for orthotropic plates under thermal conditions. In their work, the acoustic radiation effect of orthotropic plates was also discussed. Indeed, the formation of cracks can seriously affect the intrinsic property of structure elements, inducing the loss of stability. Hence, the understanding of vibration characteristics of cracked rectangular plates in thermal environment is another crucial topic. In the literature, only a limited number of studies have reported for the vibration analysis of cracked rectangular plates under thermal effects. Sahin et al. [29] applied the finite element method to investigate the thermal buckling behavior of anti-symmetric composite square plates. Natarajan et al. [30] presented a study for the free vibration of cracked functionally graded plates under thermal effects. They discussed the influence of various parameters, including crack length, aspect ratio, temperature effect and boundary condition, on functionally graded plates. Rahimabadi et al. [31] studied the thermal effect on the vibration of functionally graded material plates with the presence of a centrally located circular crack. In addition, Joshi et al. [32,33] proposed analytical models for the vibration analysis of cracked thin rectangular plates under thermal effects.

The present research focuses on the vibration and buckling analysis of isotropic and orthotropic thin rectangular plates with crack defects in thermal environment, it aims to develop an accurate and efficient solution for the prediction of structural responses by using the discrete singular convolution (DSC) method. This numerical approach was originally proposed by Wei [34,35] for solving the Fokker-Planck equation. According to the previous works [36,37], the DSC method is more stable than the differential quadrature method in the prediction of thousands of vibration modes of beams and plates. Although the DSC method is regarded as a local spectral method to possess the ability for handling complex geometries and boundary conditions, it also performs as a global numerical method with a high level of accuracy [38]. Wei et al. [39] firstly utilized the DSC method to analyze plate structures with internal supports. Lai and Xiang [40,41] employed the DSC method for solving buckling and vibration problems of transversely supported and standing vertical rectangular plates. To extend the applicability of the DSC method, the DSC-Ritz method was proposed for analyzing the vibration of thick plates and shallow shells [42–44]. Civalek [45–48] further explored the DSC approach for solving a variety of laminated shell and plate problems. Wang and his colleagues [49–54] provided plentiful research works using the DSC method for various structure problems, e.g., multiple-stepped beams. On the basis of the DSC approach, Xiang et al. [55,56] also developed the DSC-Element method to investigate the dynamic characteristics of Mindlin plates with mixed edge supports and skew angles. Since the original version of the DSC approach is restricted for dealing with vibration and buckling analysis of plates with free edges, Wang et al. [57,58] incorporated the DSC method with the Taylor series expansion technique to pass through this bottleneck.

Although there are many merits of the DSC method, to the best of the authors' knowledge, the DSC method has not been applied to investigate the vibration analysis of isotropic and orthotropic plates with crack defects in thermal environment. This research offers a comprehensive study for the vibration and buckling analysis of cracked plates under various boundary and thermal conditions. A special restrained manner of simply supported conditions that are permissible for in-plane movements under thermal conditions is also investigated. To go beyond the limitation of the original DSC approach, the Taylor series expansion method [57,58] is incorporated to address the free vibration problem of plates with free boundary constraints. To formulate an eigenvalue problem, a small matrix-band of the DSC method is employed herein. This can also enhance the computational effort in dealing with large-scale structural calculations. The results are compared with the available solutions to validate its accuracy and reliability. Besides, accurate first-known solutions are also presented in this paper.