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Inelastic condensed dynamic models for estimating seismic demands for buildings

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ABSTRACT

Computationally-efficient simulations of structural responses, such as displacements and inter-story drift ratios, are central to performance-based earthquake engineering. Calculating these responses involves potentially time-consuming response history analysis of inelastic structural behavior. To overcome this burden, this paper introduces a new inelastic model condensation (IMC) procedure. The method presented here is non-iterative and uses the modal properties of the full model (in the elastic range) to condense the structural model such that the condensed elastic model preserves the modal properties of the full model at certain modes specified by the analyst. Then, by replacing the inter-story elastic forces with hysteretic forces, the inelastic behavior of the full finite element model is incorporated into the condensed model. The parameters of these hysteretic forces are easily tuned, in order to fit the inelastic behavior of the condensed structure to that of the full model under a variety of simple loading scenarios. The fidelity of structural models condensed in this way is demonstrated via simulation for different ground motion intensities on three different building structures with various heights. The simplicity, accuracy, and efficiency of this approach could significantly alleviate the computational burden of performance-based earthquake engineering.

1. Introduction

While nonlinear response history analysis (NLRHA) is the most rigorous procedure to estimate seismic demand parameters [1, §16.2], it is computationally expensive. This is true, especially, when there is a need to perform that analysis for a large quantity of scenarios such as required for surrogate modeling, optimization, or the reliability analysis and design of structures [2–8]. Considering the time-consuming nature of detailed time-history analyses of high-dimensional inelastic models, an accurate and computationally-efficient method of predicting responses of inelastic structures, e.g. displacements and inter-story drift ratios, would be of significant value in performance-based earthquake engineering (PBEE) [9].

As alternatives to NLRHA, methods involving pushover analyses alone or in combination with inelastic response spectra are well developed [10–13]. The dynamics of the problem are captured only by the response spectrum, which reflects the dynamics of inelastic single degree of freedom (DOF) systems. The mechanics of the detailed structural model is reflected only by the pushover analysis, which does not involve dynamics and does require the specification of a distribution of lateral static loads. Several studies have focused on the specification of lateral load distributions for pushover analyses, such as Refs. [14–18], among others. Unfortunately, simple pushover analyses neglect the effect of higher modes on the response. To tackle this, a few notable studies incorporate the higher modes' dynamics in pushover analyses [19–21]. Nevertheless, the errors in the predictions of displacements and drift ratios are in the order of 30% [19].

Other researchers have approached this problem using reduced (condensed) structural models [7,8,22]. This trend started when the static and dynamic condensations were introduced [23,24], and it continues to be refined [25,26]. Previous methods of model reduction for hysteretic structures have either been limited to reducing only the linear aspects of the system [27], retaining all the nonlinear elements present in the system at some computational expense [28,29], or to approximating the nonlinear system using modal superposition with time-varying modes [30,31]. Recently, researchers are developing and evaluating model reductions for modeling the hysteretic behavior of structures [7], identifying damage detection [27,32], modeling elastomers [33], and glass structures [34].

In this paper, a new condensation method in conjunction with a framework for application is proposed for condensation of inelastic dynamic structural models, by using the modal properties and replacing

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the elastic restoring forces with the hysteretic forces. In the following, the inelastic model condensation (IMC) procedure to construct the linear condensed model is first described. Next, the procedure to extend the linear condensed model with hysteretic (Bouc–Wen) elements is presented and the optimal values of the hysteretic parameters are obtained using the Levenberg–Marquardt algorithm for several types of loading regimes. Next, the resulting inelastic condensed models with parameters fit with those different regimes are evaluated by imposing a real earthquake to both the nonlinear finite element model (NFEM) and the condensed model. Finally, the paper concludes with a discussion of the results and highlights the performance of the proposed IMC approach.

2. Inelastic condensed dynamic modeling procedure

In the reduced-order modeling method presented here, the reduced linear model is derived to match the natural frequencies and mode-shapes of the *full* model at selected modes. The coordinates of the *condensed* model correspond to the selected coordinates of the full model. Then, the elastic restoring forces of the linear condensed model are simply replaced by hysteretic forces. The hysteretic forces are evolutionary [35] and are calibrated to match the inelastic behavior of the detailed inelastic frame (*NFEM*) model. Importantly, the number of hysteretic variables need not be larger than the number of condensed coordinates and time-varying (or 'nonlinear') modes are not involved.

2.1. Full model

Consider a planar frame (structure) subject to horizontal ground motion $\ddot{u}_g(t)$. The set of linear equations that define the n-DOF structural system is given by:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = -\mathbf{M}\iota\ddot{u}_g \tag{1}$$

where $\mathbf{q}$ is the n-dimensional vector of DOFs, which may include lateral displacements, vertical displacements, and rotations; $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the $n \times n$ mass, damping, and (linear-elastic) stiffness matrices, respectively; and ι is the n-dimensional influence vector that applies $\ddot{u}_g$ to the lateral nodal displacements.

2.2. Condensed model

First, the full model (Eq. (1)) is reduced to a *condensed* model with fewer DOFs, say r DOFs, using the *mass orthonormal* mode shapes found from the full model, $\mathbf{\Phi} = [\boldsymbol{\phi}_1, ..., \boldsymbol{\phi}_n]$. The mode shapes $\boldsymbol{\phi}_i$ should be sorted by the absolute value of the modal participation factors $\Gamma_i \equiv \boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\iota$ (or equivalently the modal participating mass ratio), as opposed to sorting based on the frequency (lowest to highest), to emphasize the highest contributing modes to the seismic response.

Let $\mathbf{S} \in \mathbb{R}^{r\times n}$ be the *selection matrix* that specifies the DOFs to be retained, denoted $\mathbf{u} \subset \mathbf{q}$; i.e., $\mathbf{u} = \mathbf{S}\mathbf{q}$. Generally, the retained coordinates $\mathbf{u}$ can be taken to be any set of r DOFs. However, in order to introduce the nonlinear behavior in the present application, only horizontal floor displacements should be selected, at most one per floor. In this study, all the stories are included, though this need not be the case in general.

The mode shapes in the retained DOFs are defined as $\boldsymbol{\psi}_i = \mathbf{S}\boldsymbol{\phi}_i$ $(i = 1, ..., r)$. Now, the reduced model is reconstructed directly from the reduced selected mode shapes. That is:

$$\begin{aligned}
&\mathbf{\Psi}^{\mathrm{T}}\mathcal{M}\mathbf{\Psi} = \mathbf{I}_{r\times r},\\
&\mathbf{\Psi}^{\mathrm{T}}\mathcal{C}\mathbf{\Psi} = \mathbf{diag}(2\zeta_1\omega_1, ..., 2\zeta_r\omega_r),\\
&\mathbf{\Psi}^{\mathrm{T}}\mathcal{K}\mathbf{\Psi} = \mathbf{diag}(\omega_1^2, ..., \omega_r^2)
\end{aligned} \tag{2}$$

where $\mathcal{M}$, $\mathcal{C}$, and $\mathcal{K}$ are the reduced mass, damping, and stiffness matrices; the square matrix $\mathbf{\Psi} = [\boldsymbol{\psi}_1, ..., \boldsymbol{\psi}_r]$; and ω_i and ζ_i are the natural frequency and damping ratio in the ith mode of the full model. Then, the reduced model may be written as:

$$\mathcal{M}\ddot{\mathbf{u}} + \mathcal{C}\dot{\mathbf{u}} + \mathcal{K}\mathbf{u} = -\mathbf{R}\ddot{u}_g \tag{3}$$

where $\mathbf{\Psi}^{\mathrm{T}}\mathbf{R} = [\Gamma_1, ..., \Gamma_r]$.

Unlike some reduced models that only match the dominant mode of the full model [7], this condensed model (Eq. (3)) exactly matches the modal dynamics of the full model at the selected frequencies $(\omega_1, ..., \omega_r)$. The condensed model can be thought of as a stick model with communication between all the stories, i.e., $\mathcal{M}$, $\mathcal{C}$, and $\mathcal{K}$ are fully populated, in general.

2.3. Treatment of hysteresis

Inelastic restoring forces are then incorporated into the condensed model (Eq. (3)) by replacing the inter-story forces with inelastic restoring forces. The lateral deflections $\mathbf{u}$ relative to the ground are related to the inter-story deflections, denoted by $\mathbf{\Delta} = [\Delta_1, ..., \Delta_r]^{\mathrm{T}}$, through the linear transformation:

$$\mathbf{u} = \mathbf{L}\mathbf{\Delta} \;\Rightarrow \mathbf{\Delta} = \mathbf{L}^{-1}\mathbf{u} \tag{4}$$

where $\mathbf{L}$ is a r-dimensional lower-triangular matrix of unity. The (elastic) inter-story forces are therefore given by $\mathbf{L}^{\mathrm{T}}\mathcal{K}\mathbf{L}\mathbf{\Delta}$. Now, instead of elastic restoring forces, the inter-story shear forces are taken to be:

$$\mathbf{L}^{\mathrm{T}}\mathcal{K}\mathbf{L}\mathbf{\Delta} \rightarrow \boldsymbol{\kappa}\mathbf{L}^{\mathrm{T}}\mathcal{K}\mathbf{L}\mathbf{\Delta} + (\mathbf{I}-\boldsymbol{\kappa})\mathbf{L}^{\mathrm{T}}\mathcal{K}\mathbf{L}\mathbf{z} \tag{5}$$

in which $\boldsymbol{\kappa}$ is a diagonal matrix where each element κ_i is the ratio of the post-yield stiffness to pre-yield (elastic) stiffness at the ith story. The auxiliary variables $\mathbf{z} = [z_1, ..., z_r]^{\mathrm{T}}$ are the hysteretic displacements, which are given by the Bouc–Wen model [36,37]:

$$\dot{z}_i = \dot{\Delta}_i - \beta_i\,|\dot{\Delta}_i|\,|z_i|^{\eta_i-1}z_i - \gamma_i\dot{\Delta}_i|z_i|^{\eta_i}. \tag{6}$$

The uniaxial hysteretic behavior at the ith story is governed by the hysteretic parameters η_i, β_i, and γ_i, independent of the other stories. The parameter η_i governs the smoothness of the transition from the linear to the nonlinear range [38], effectively adjusting the 'knee' of the hysteretic loop. The parameters β_i and γ_i govern the isotropic yield displacement $z_{i,\mathrm{yield}}$ in the ith story as follows:

$$z_{i,\mathrm{yield}} = (\beta_i + \gamma_i)^{-\frac{1}{\eta_i}}. \tag{7}$$

Inversely, if the yield displacement $z_{i,\mathrm{yield}}$ is prescribed, then the hysteretic parameters are:

$$\beta_i = \frac{\rho_i}{z_{i,\mathrm{yield}}^{\eta_i}} \quad \text{and} \quad \gamma_i = \frac{1-\rho_i}{z_{i,\mathrm{yield}}^{\eta_i}} \tag{8}$$

where $0 \leqslant \rho_i \leqslant 1$. By varying ρ_i, the hysteretic loop shape changes, as illustrated in Fig. 1. (Note that other simplified hysteretic models could

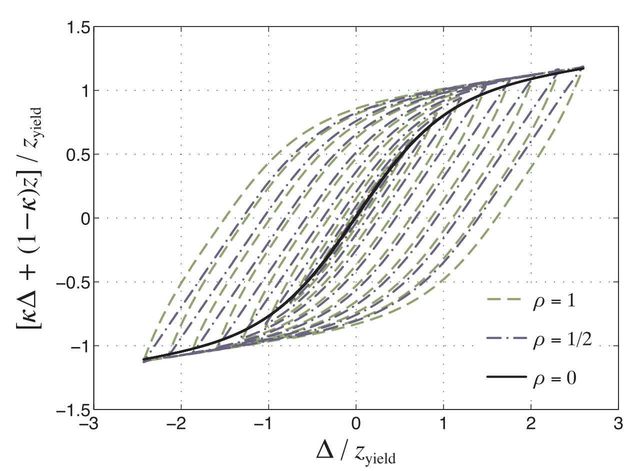

Fig. 1. Representative hysteresis loops for $\kappa = 0.1$ and $\eta = 2$.