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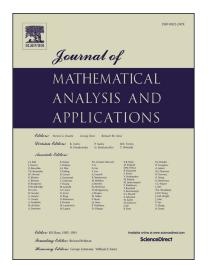
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ACCEPTED MANUSCRIPT

Homogenization of some degenerate pseudoparabolic variational inequalities

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Abstract

Multiscale analysis of a degenerate pseudoparabolic variational inequality, modelling the two-phase flow with dynamical capillary pressure in a perforated domain, is the main topic of this work. Regularisation and penalty operator methods are applied to show the existence of a solution of the nonlinear degenerate pseudoparabolic variational inequality defined in a domain with microscopic perforations, as well as to derive a priori estimates for solutions of the microscopic problem. The main challenge is the derivation of a priori estimates for solutions of the variational inequality, uniformly with respect to the regularisation parameter and to the small parameter defining the scale of the microstructure. The method of two-scale convergence is used to derive the corresponding macroscopic obstacle problem.

Keywords: pseudoparabolic inequalitues, obstacle problems, degenerate nonlinear PDEs, homogenization, two-scale convergence, penalty operator method 2010 MSC: 35B27, 35K65, 35K70, 35K86, 35Q35, 76S05

1. Introduction

In this paper we consider multiscale analysis of a nonlinear degenerate pseudoparabolic variational inequality modelling unsaturated flow with dynamic capillary pressure in a perforated porous medium. Models for two-phase flow with dynamical capillary pressure, originally proposed by [17, 38], consider Darcy's law for the flux of the moisture content u given by

$$J = -A k(u) (\nabla p + \mathbf{e}_n),$$

and assume that the pressure p in the wetting phase is a function of the moisture content u and its time derivative $\partial_t u$, i.e. in a simplified form,

$$p = -\tilde{P}_c(u) + \tau \partial_t u,$$

where the permeability function k(u) depends on the moisture content, the vector $\mathbf{e}_n = (0, \dots, 0, 1)$ determines the direction of flow due to gravity, and A and τ are positive constants. Then for the moisture content u we obtain a pseudoparabolic equation of the from

$$\partial_t u = \nabla \cdot \left(A \, k(u) [P_c(u) \nabla u + \tau \nabla \partial_t u + \mathbf{e}_n] \right), \tag{1}$$

where $P_c(u) = -\tilde{P}'_c(u)$.

If considering a two-phase flow problem in a perforated porous medium with Signorini's type conditions on the surfaces of perforations

$$u \ge 0, \quad A k(u) (P_c(u) \nabla u + \tau \nabla \partial_t u + \mathbf{e}_n) \cdot \nu \ge -f(t, x, u), u [A k(u) (P_c(u) \nabla u + \tau \nabla \partial_t u + \mathbf{e}_n) \cdot \nu + f(t, x, u)] = 0,$$
(2)

then a weak formulation of equation (1) together with conditions (2) results in a pseudoparabolic variational inequality of the form

$$\langle \partial_t u, v - u \rangle_{G^{\varepsilon}} + \langle A \, k(u) [P_c(u) \nabla u + \tau \partial_t \nabla u + \mathbf{e}_n], \nabla (v - u) \rangle_{G^{\varepsilon}} + \langle f(t, x, u), v - u \rangle_{\Gamma^{\varepsilon}} \ge 0, \tag{3}$$

where $G^{\varepsilon} \subset \mathbb{R}^n$, with n = 2, 3, denotes the perforated domain and Γ^{ε} defines the boundaries of perforations.

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