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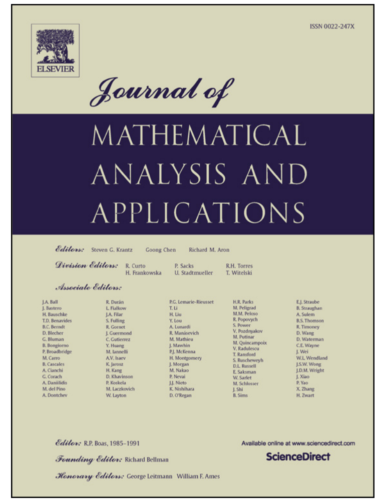


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# Homogenization of some degenerate pseudoparabolic variational inequalities

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**Abstract**

Multiscale analysis of a degenerate pseudoparabolic variational inequality, modelling the two-phase flow with dynamical capillary pressure in a perforated domain, is the main topic of this work. Regularisation and penalty operator methods are applied to show the existence of a solution of the nonlinear degenerate pseudoparabolic variational inequality defined in a domain with microscopic perforations, as well as to derive a priori estimates for solutions of the microscopic problem. The main challenge is the derivation of a priori estimates for solutions of the variational inequality, uniformly with respect to the regularisation parameter and to the small parameter defining the scale of the microstructure. The method of two-scale convergence is used to derive the corresponding macroscopic obstacle problem.

*Keywords:* pseudoparabolic inequalitues, obstacle problems, degenerate nonlinear PDEs, homogenization, two-scale convergence, penalty operator method
*2010 MSC:* 35B27, 35K65, 35K70, 35K86, 35Q35, 76S05

## 1. Introduction

In this paper we consider multiscale analysis of a nonlinear degenerate pseudoparabolic variational inequality modelling unsaturated flow with dynamic capillary pressure in a perforated porous medium. Models for two-phase flow with dynamical capillary pressure, originally proposed by [17, 38], consider Darcy's law for the flux of the moisture content $u$ given by

$$J = -A\,k(u)(\nabla p + \mathbf{e}_n),$$

and assume that the pressure $p$ in the wetting phase is a function of the moisture content $u$ and its time derivative $\partial_t u$, i.e. in a simplified form,

$$p = -\tilde{P}_c(u) + \tau\partial_t u,$$

where the permeability function $k(u)$ depends on the moisture content, the vector $\mathbf{e}_n = (0, \dots, 0, 1)$ determines the direction of flow due to gravity, and $A$ and $\tau$ are positive constants. Then for the moisture content $u$ we obtain a pseudoparabolic equation of the from

$$\partial_t u = \nabla \cdot \big(A\,k(u)[P_c(u)\nabla u + \tau\nabla\partial_t u + \mathbf{e}_n]\big), \tag{1}$$

where $P_c(u) = -\tilde{P}_c'(u)$.

If considering a two-phase flow problem in a perforated porous medium with Signorini's type conditions on the surfaces of perforations

$$\begin{aligned} &u \geq 0, \quad A\,k(u)(P_c(u)\nabla u + \tau\nabla\partial_t u + \mathbf{e}_n)\cdot\nu \geq -f(t,x,u), \\ &u\big[A\,k(u)(P_c(u)\nabla u + \tau\nabla\partial_t u + \mathbf{e}_n)\cdot\nu + f(t,x,u)\big] = 0, \end{aligned} \tag{2}$$

then a weak formulation of equation (1) together with conditions (2) results in a pseudoparabolic variational inequality of the form

$$\langle \partial_t u, v-u\rangle_{G^\varepsilon} + \langle A\,k(u)[P_c(u)\nabla u + \tau\partial_t\nabla u + \mathbf{e}_n], \nabla(v-u)\rangle_{G^\varepsilon} + \langle f(t,x,u), v-u\rangle_{\Gamma^\varepsilon} \geq 0, \tag{3}$$

where $G^\varepsilon \subset \mathbb{R}^n$, with $n = 2, 3$, denotes the perforated domain and $\Gamma^\varepsilon$ defines the boundaries of perforations.

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