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Quasineutral limit of the two-fluid Euler–Poisson system in a bounded domain of $\mathbb{R}^3$

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ABSTRACT

The quasineutral limit of the two-fluid Euler–Poisson system (one for ions and another for electrons) in a bounded domain of $\mathbb{R}^3$ is rigorously proved by investigating the existence and the stability of boundary layers. The non-penetration boundary condition for velocities and Dirichlet boundary condition for electric potential are considered.

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1. Introduction

One is usually concerned with large scale structures with respect to the Debye length λ_D (typically $\lambda_D \approx 10^{-5}$–$10^{-8}$) in plasma. For such scales, the plasma can be considered as being almost electrically neutral. However, in a domain with boundaries, the quasineutrality breaks down near the boundary since the boundary layers generally develop due to the interaction between plasma and the boundary. The mathematical analysis of this phenomena of boundary layers has attracted much attention. Brezis, Golse and Sentis [2], and Ambroso, Mehats and Raviart [1] performed the boundary analysis when the Poisson equation is considered alone (the density of ions being prescribed). Slemrod and Sternberg [18] proved rigorously the quasineutral limit for some stationary Euler–Poisson system (with massless electrons) on a bounded domain in one dimension. Peng–Wang [13] and Violet [19] studied the quasineutral limit of some Euler–Poisson system (with fixed ions) for stationary and irrotational flows on a bounded domain of $\mathbb{R}^d$ $(d = 2, 3)$. Recently, the quasineutral limit was proved rigorously for nonstationary multidimensional Euler–Poisson

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system for ions (the electrons following the classical Maxwell–Boltzmann relation: $n = e^{-\phi}$) in a domain with boundaries by Gérard-Varet, Han-Kwan and Rousset in [5,6].

In this paper, our goal is to study rigorously the quasineutral limit for two-fluid Euler–Poisson system (one for ions and another for electrons) in a bounded domain of $\mathbb{R}^3$. We consider the following dimensionless two-fluid Euler–Poisson system, for $t > 0, x = (x_1, x_2, x_3) = (y, x_3) \in \mathbb{T}^2 \times (0, 1)$,

$$\partial_t n^\lambda + \mathrm{div}(n^\lambda u^\lambda) = 0, \tag{1.1}$$

$$\partial_t u^\lambda + u^\lambda \cdot \nabla u^\lambda + T^e \nabla \ln(n^\lambda) = -\nabla\phi^\lambda, \tag{1.2}$$

$$\partial_t h^\lambda + \mathrm{div}(h^\lambda v^\lambda) = 0, \tag{1.3}$$

$$\partial_t v^\lambda + v^\lambda \cdot \nabla v^\lambda + T^i \nabla \ln(h^\lambda) = \nabla\phi^\lambda, \tag{1.4}$$

$$\lambda^2 \Delta\phi^\lambda = h^\lambda - n^\lambda, \tag{1.5}$$

where n^λ is the density of electron, h^λ is the density of ions, u^λ is the velocity of electron, v^λ is the velocity of ions, and ϕ^λ is the electric potential. $\lambda = \lambda_D/L \ll 1$ with L being the characteristic length. T^e and T^i are the average temperature of electrons and ions, respectively and $T^e >> T^i$. We consider the following boundary conditions

$$u_3^\lambda|_{x_3=0,1} = 0, \quad v_3^\lambda|_{x_3=0,1} = 0, \quad \phi^\lambda|_{x_3=0,1} = \phi_b, \tag{1.6}$$

where $\phi_b = \phi_{\mathrm{ref}} + \phi(y)$ with ϕ_{ref} being a constant and $\phi(y)$ is a smooth function. Formally, as $\lambda \to 0$, the limit $(n, h, u, v, \phi)(t, x)$ of $(n^\lambda, h^\lambda, u^\lambda, v^\lambda, \phi^\lambda)(t, x)$ satisfies the system

$$\partial_t n + \mathrm{div}(nu) = 0, \tag{1.7}$$

$$\partial_t u + u \cdot \nabla u + T^e \nabla \ln n = -\nabla\phi, \tag{1.8}$$

$$\partial_t h + \mathrm{div}(hv) = 0, \tag{1.9}$$

$$\partial_t v + v \cdot \nabla v + T^i \nabla \ln h = \nabla\phi, \tag{1.10}$$

$$n = h, \tag{1.11}$$

and we denote $n = h =: m$. When $(n, u)(0, x) = (h, v)(0, x)$, we can further prove that $u(t, x) = v(t, x) =: w(t, x)$ for all $t \in [0, T]$ for some $T > 0$ (see section 2). Then adding the momentum equations (1.8) and (1.10), we see that (m, w) satisfies one-fluid compressible isothermal Euler equations

$$\partial_t m + \mathrm{div}(mw) = 0, \tag{1.12}$$

$$\partial_t w + w \cdot \nabla w + \frac{T^e + T^i}{2} \nabla \ln(m) = 0, \tag{1.13}$$

with

$$\phi = \frac{T^i - T^e}{2} \ln m + f(t). \tag{1.14}$$

It is natural to consider the convergence of (1.1)–(1.5) to the system (1.12)–(1.14) as $\lambda \to 0$. When the model is considered in a domain without boundary, a formal asymptotic expansion was first introduced by Peng and Wang [16], and the justification of this expansion was validated by Ju et al. in [9]. The corresponding results for nonisentropic two-fluid Euler–Poisson system were obtained in [8]. By exploring the coupling of Euler equations and the Poisson equation, Li, Peng and Wang [11] improved the convergence rate obtained

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