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Stability of attractors for the Kirchhoff wave equation with strong damping and critical nonlinearities ☆

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ABSTRACT

The paper investigates longtime dynamics of the Kirchhoff wave equation with strong damping and critical nonlinearities: $u_{tt} - (1 + \epsilon\|\nabla u\|^2)\Delta u - \Delta u_t + h(u_t) + g(u) = f(x)$, with $\epsilon \in [0, 1]$. The well-posedness and the existence of global and exponential attractors are established, and the stability of the attractors on the perturbation parameter ϵ is proved for the IBVP of the equation provided that both nonlinearities $h(s)$ and $g(s)$ are of critical growth.

1. Introduction

In this paper, we are concerned with the existence and stability of global and exponential attractors for the Kirchhoff wave equation with strong damping and critical nonlinearities

$$u_{tt} - (1 + \epsilon\|\nabla u\|^2)\Delta u - \Delta u_t + h(u_t) + g(u) = f(x) \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{1.1}$$

$$u|_{\partial\Omega} = 0, \quad u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega, \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$ with the smooth boundary $\partial\Omega$, $\epsilon \in [0, 1]$, $h(s)$ and $g(s)$ are nonlinear functions specified later, and $f(x)$ is an external force term.

In one space dimension case, Eq. (1.1), without dissipation $-\Delta u_t$ and nonlinear perturbations $h(u_t)$ and $g(u)$, was introduced by Kirchhoff [19] to describe the oscillation of an elastic, stretched string. The strong damping $-\Delta u_t$ naturally arises if, for instance, the string is made up of the viscoelastic material of

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rate-type [13], and it indicates that the stress is proportional not only to strain but also to strain rate as in a linearized Kelvin–Voigt material. There have been extensive researches on the global existence and the decay properties of solutions for the Kirchhoff wave equation with different type of dissipations (see, e.g. [1,2,8,16,17,21,22,25–27] and references therein).

When $\epsilon = 0$, Eq. (1.1) is reduced to the following strongly damped semi-linear wave equation

$$u_{tt} - \Delta u_t - \Delta u + h(u_t) + g(u) = f, \tag{1.3}$$

which governs the thermal evolution in a homogeneous isotropic rigid body, where $h(u_t)$ denotes a nonlinearly temperature-dependent internal source term, and $g(u)$ denotes a source term depending nonlinearly on displacement (cf. Appendix B in [10]).

Global attractor and exponential attractor are two key concepts to study the asymptotic behavior of solutions for the nonlinear evolution equations with various dissipations. When $\Omega \subset \mathbb{R}^3$, the growth exponents q and p of the nonlinearities $h(u_t)$ and $g(u)$ appearing in Eq. (1.3) are (at most) critical, that is, $1 \leq q, p \leq 5$, a complete analysis on the existence of global and exponential attractors of optimal regularity for problem (1.3)–(1.2) has been done by Dell'Oro and Pata in [9–11]. For the related researches on this topic, one can see [5,15] and references therein. In this category, the critical nonlinearity is especially concerned in mathematics because in this case the compactness of the Sobolev embedding fails.

Recently, for general bounded smooth domain $\Omega \subset \mathbb{R}^N$, with $N \geq 3$, when the growth exponents q and p are fully supercritical, that is, $q = p > p^* = \frac{N+2}{N-2}$ (in this case, one can not get the uniqueness of the weak solutions), Yang and Liu [34] further established the existence of global attractor for the subclass of limit solutions of problem (1.3)–(1.2) by using J. Ball's attractor theory on the generalized semiflow. By the way, here the exponent $p^* \equiv \frac{N+2}{N-2}$ ($N \geq 3$) is called critical relative to the energy space $H_0^1(\Omega) \times L^2(\Omega)$ because $H_0^1 \hookrightarrow L^{p+1}$ for $p \leq p^*$ but the embedding fails for $p > p^*$.

But when $\epsilon > 0$, the appearance of the Kirchhoff nonlocal nonlinear term causes some essential difficulties, which leads to that the techniques used to deal with the semi-linear wave equation (1.3) as done in [9–11,34] cease to be effective.

In comparison with extensive researches on the longtime dynamics of semilinear wave equations there are relatively few results on that of the quasi-linear wave equations. For the researches on this topic, one can see [3,14,18,20,23,24,31,32,36] and references therein. Recently, Chueshov [3] studied the well-posedness and longtime dynamics for the Kirchhoff wave model with strong nonlinear damping

$$u_{tt} - \phi(\|\nabla u\|^2)\Delta u - \sigma(\|\nabla u\|^2)\Delta u_t + g(u) = f(x). \tag{1.4}$$

A main breakthrough is that he found a supercritical exponent $p^{**} \equiv \frac{N+4}{(N-4)^+}$, where $a^+ = \max\{a, 0\}$, and showed that when the growth exponent p of the source term $g(u)$ is up to the supercritical range: $p^* < p < p^{**}$, problem (1.4)–(1.2) is still well-posed and the related evolution semigroup has a finite-dimensional global attractor in phase space $\mathcal{H} = H_0^1 \cap L^{p+1} \times L^2$ in the sense of "partially strong topology". More recently, Ding, Yang and Li [12] further removed the restriction of "partially strong topology" in [3] and established the existence of finite-dimensional global attractor in the natural energy space. Ma and Zhong [20] showed that the global attractor in phase space $\mathcal{H} = H_0^1 \times L^2$ are of $H_0^1 \times H_0^1$-compactness and $H_0^1 \times H_0^1$-attractiveness.

But the appearance of the nonlinear perturbation $h(u_t)$ in Eq. (1.1) causes that the methods used to treat Kirchhoff wave model (1.4) as in [3,12,20] become useless. Especially in the situation that the growth of both nonlinearities $h(s)$ and $g(s)$ are up to the critical range, i.e., $1 \leq q, p \leq p^*$, what about the well-posedness and the longtime behavior of the solutions of problem (1.1)–(1.2)? what about the stability of the global and exponential attractors on the perturbation parameter ϵ (especially as $\epsilon \to 0$)? These questions are still unsolved.

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