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Effects of Lévy noise on transport of active Brownian particles

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HIGHLIGHTS

- The transport properties of active Brownian particles driven by Lévy noise are studied.
- Performance is strongly sensitive to Lévy index α and noise strength of Lévy noise.
- The optimal value of Lévy index α for which the mean positive velocity shows minimum may be important for other investigations.

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ABSTRACT

The transport properties of active Brownian particles driven by Lévy noise are investigated. It is found the Lévy index α plays a significant role in the transport of the active Brownian particles, where both the mean velocity and the mean escape time show the non-monotonic dependence on the Lévy index. The effective diffusion increases as the Lévy index α increases.

1. Introduction

During the past two decades, the constructive role of noise has been widely observed in physical, chemical, as well as in biological systems. Examples manifesting constructive role of noises include stochastic resonance [1–3], spatiotemporal order out of noise [4], and noise-induced transport [5,6], etc. The transport properties of Brownian particles affected by noise have been investigated extensively due to the possible applications in various fields, such as molecular motors [7–11], nanoscale friction [12,13], surface smoothening [14], and so on. In the simplest situations, it is assumed that the noise acting on a test particle is white and Gaussian. In far from equilibrium situations, the stochastic process describing interactions of the test particle with the environment can still be of the white type but its values can be distributed according to some heavy tailed distribution. Lévy flight, which denotes a class of non-Gaussian stochastic processes, can be observed in many different situations ranging from the detection of neurons faint or subthreshold signals [15], self-diffusion in micelle systems [16–18], super-diffusion in a magnetic field [19], genetic regulatory [20], and many others [21–23]. Due to heavy-tail characteristics and infinite divisibility, the dynamical systems subjected to the Lévy noise exhibit quite unusual and fascinating properties. Among these studies, one important issue is to explore the non-equilibrium effect of the Lévy noise in the Lévy ratchet. J.D. Bao pointed out that the presence of α-stable Lévy type noise affects the escape process of the particles and the escape rate shows nonmonotonic dependence on the Lévy noise [24]. Dybiec and co-workers [25,26] studied the minimal setup for a Lévy ratchet and found that due to the nonthermal character of the Lévy noise, the net current can be obtained even in the absence of whatever additional time-dependent forces. Rosa and Beims [27] studied the optimal transport and its

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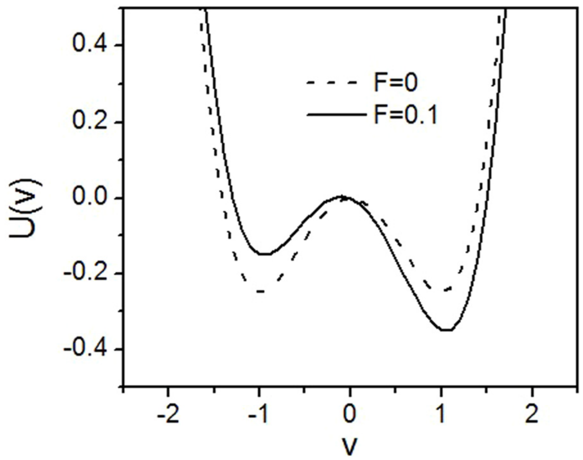

Fig. 1. The velocity potential of the active Brownian particles for the symmetric case (F = (0) and the asymmetric case (F = 0.1)

relation to superdiffusive transport and Lévy walks for Brownian particles in ratchet potential in the presence of modulated environment and external oscillating forces. B.Q. Ai studied the transport of Brownian particles in the presence of ac-driving forces and Lévy noises, and multiple current reversals were observed [28].

Recently, the transport properties of active Brownian particles (ABP) have attracted much attention [29,30] since it can explain the origin of the self-propelled motion, and the active(self-propelled) motion is one of the obvious signatures of many living systems and can be observed on different levels, ranging from the motion of motor proteins within cells to the motility of cells and bacteria [31–33]. Our system is closely related to the one investigated in Ref. [34], where the mean drift and effective diffusion of such a system are discussed when a simple Gaussian noise is applied. However, Lévy noise has advantages over standard Gaussian noise in many models though it increases mathematical complexity. Therefore, the effects of the Lévy noise on the transport properties of the active Brownian particles need to be further investigated.

In this paper, the transport properties of the active Brownian particle driven by Lévy stable noise are investigated. In Section 2, the dynamical model of the active Brownian particles with Lévy noise is presented. In Section 3, the effects of Lévy noise on the transport of active Brownian particles are discussed. A discussion concludes the paper.

2. Dynamical model and theoretical analysis

The Langevin equation of the active Brownian particle driven by Lévy stable noise can be written as follows [35–37]

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -\frac{dU(v)}{dv} + \xi_\alpha(t) \end{aligned} \tag{1}$$

Here all parameters and variables in Eq. (1) are dimensionless. $x(t)$ is the position of the particle at time t. The biased velocity potential $\mathrm{U}(v) = v^4/4 - v^2/2 - Fv$ is plotted in Fig. 1 where the asymmetry is introduced by a bias F. It is shown that the potential is symmetric for F = 0, and asymmetric for F = 0.1. Since the influence of the bias F on the transport of active Brownian particles has already been reported in Ref. [34], in the following discussion F is fixed to be 0.1 for a non-zero velocity. The Lévy noise $\xi_\alpha(t)$ is a white, symmetric α-stable noise possessing Lévy stable distribution $\lambda(x) \sim \frac{\sigma^\alpha}{|x|^{1+\alpha}}$ with $0 < \alpha \leq 2$, where σ is a scaling factor of physical dimension length $[\sigma] = \mathrm{cm}$. [38] In the Fourier domain, the characteristic function $\lambda(k)$ of $\lambda(x)$ can be written as [39,40]

$$\lambda(k) = F\{\lambda(x)\} = \int_{-\infty}^{\infty} e^{ikx}\lambda(x)\,dx = exp\left(-D|k|^\alpha\right). \tag{2}$$

Here $\mathrm{D} = \sigma^\alpha$ denotes the strength of noise [41].

The simulation strategy of Eq. (1) can be given as [42]

$$\begin{aligned} x_{n+1} - x_n &= v_n \\ v_{n+1} - v_n &= -\frac{dU(v_n)}{dv}\Delta t + (D\Delta t)^{1/\alpha}\,\xi_\alpha(n). \end{aligned} \tag{3}$$