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Link deletion in directed complex networks

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HIGHLIGHTS

- The study details the response and robustness of directed networks to loss of links.
- It includes results on both standard network models as well as three real networks.
- The strategies for removal of links are based on global and local network properties.
- Responses are characterized by the extent and efficiency of structural connectivity.
- The specific roles of clustering and 2-node degree correlations are also explored.

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ABSTRACT

We present a systematic and detailed study of the robustness of directed networks under random and targeted removal of links. We work with a set of network models of random and scale free type, generated with specific features of clustering and assortativity. Various strategies like random deletion of links, or deletions based on betweenness centrality and degrees of source and target nodes, are used to breakdown the networks. The robustness of the networks to the sustained loss of links is studied in terms of the sizes of the connected components and the inverse path lengths. The effects of clustering and 2-node degree correlations, on the robustness to attack, are also explored. We provide specific illustrations of our study on three real-world networks constructed from protein–protein interactions and from transport data.

1. Introduction

The field of complex systems has caught the interest of the academic community and the industry in a big way in the recent past. It deals with how locally interacting constituents give rise to macroscopic structures and emergent phenomena. The study of these inter-dependencies is most effective with a complex network based approach. A comprehensive study of these networks would involve a long list of topics including descriptive analysis, study of generative models, dynamical or functional properties and more [1–4]. In this work, we are interested in studying their robustness. It is one of the more important aspects of a complex network and is the ability of the network to withstand and (or) adapt to internal and (or) external changes. While these changes can occur in many different ways, those resulting in loss of connectivity or breakdown of the network are of particular interest. In the not-so-recent past, we have been witness to multiple events that have persistently brought up the question of robustness. Be it global internet-failures, large-scale email attacks [5], country-wide

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electricity blackouts [6–10] or international stock-market crashes [11–14], they have led us to ask the question: *When does the failure of individual constituents translate to a failure of the whole?*

The connectivity of a network can be compromised as a result of random or targeted loss of nodes or edges. While the phenomenon of node-deletion has been extensively studied and used [15–27], to a large extent in undirected networks and to a lesser extent in directed networks, the avenue of link deletion has remained rather unexplored [28–35]. Even the existing studies, concerning the robustness of directed networks [36,37], are limited to very specific contexts or systems and the utility of the results outside the respective contexts is minimal. There is a large number of real world networks, like biological, technological and transport networks, that are directed in nature. But there is no exhaustive study that covers the robustness of standard directed-network models to common methods of loss of links, in a systematic manner. Hence we find this area to be promising and worth exploring.

Link-deletion may seem similar to the process of node deletion, except that the node is still functional but unable to contribute to the overall functioning of the network. This perception changes once we realize that it is a much more fine-tuned way to manipulate the connectivity of a network. Also it leaves the node functional, which makes reconnection a possibility. Hence this process can be used to enhance the efficiency of flow on a network [32] and also to constrain the flow if need be. For example, in a power grid, losing a few transmission lines, due to natural or man-made reasons, could result in loss of power-supply. Besides lowering the efficiency of the grid, this could also destabilize other nodes or lines, due to overloading, thus leading to a cascading failure. Detaching the unstable nodes from the network by intentional and targeted link deletion, thus preventing a cascade of failures, can salvage this situation. Along similar lines, applications can also be seen in computer networks (to control the spread of a virus), disease spreading networks (prevent interactions to control the spread of a disease) and transport networks (how will a damaged road/track or a traffic jam effect the movement of traffic?). Another important application is in the study of some neuro-degenerative disorders like Alzheimer's disease, where the loss of connectivity between neurons leads to lower efficiency in transmission of signals. Link deletion can also be used to optimize information flow on a network. Apart from the above, knowledge of the behavior of a network, when it is under different types of breakdown, helps us to identify patterns of structural and functional vulnerabilities and hence design robust systems that are resilient to various kinds of attacks and also put in place, precautionary measures that would come into effect in the event of breakdown.

In this paper, we present the results of our investigations on the effects of loss of links in directed complex networks with commonly observed properties like scale-free nature, clustering and degree-correlations. We intend to explore the structural changes occurring in the networks, in the event of failures (random loss of links) and attacks (targeted loss of links). To make this study systematic and more general, we work initially with a set of network models rather than actual real networks. In doing so, we also isolate the response of networks with specific properties to various kinds of link deletion mechanisms. We do a comparative study of the behaviors of various models of directed networks under different kinds of link deletion. In the later part of the paper, we present results of various types of link deletion on data drawn from real-world networks and look at possible interpretations of the same.

The paper is organized as follows: In Section 2, we discuss the generation of directed networks with requisite properties like clustering and degree correlations and introduce the sample datasets that we have considered. In Section 3, we present the different strategies by which links are deleted. In this section, we also compare strategies based on local and global information and define the "edge-degree" in directed networks. In the following section, we identify a set of relevant measures that capture the changes occurring in the network during link-deletion. In Section 5 we present the results of each link deletion strategy on different types of networks including three real world networks with specific properties. In Section 6, we present the results followed by conclusions.

2. Generation of directed networks

Despite the wide spectrum of phenomena to which a network based approach can be directly applied, a bulk of the real-world networks exhibit some predominantly common characteristics. Thus many real networks exhibit a heavy-tailed degree distribution, leading to the famous power-law in the degree-distributions and consequent scale-free behavior. Many such networks also exhibit the property of transitivity or clustering, to varying extents. Online Social Networks (OSNs) like Facebook, Twitter, Quora etc. have high values of clustering coefficients. Other examples of highly clustered networks include human or animal brain networks, friendship networks and co-authorship networks. But most technological networks and some biological networks like protein interaction networks, gene regulatory networks and metabolic networks show very little clustering. Another important property seen in many real-networks is the correlation between nodes, based on scalar or enumerative properties. For example, most social networks have been observed with high positive correlation (Assortativity) with respect to scalar characteristics like age, sex, hobbies etc. On the other hand, networks like the World Wide Web (WWW) and the internet show negative correlations (Dissortativity). Some other properties of networks include the existence of communities and hierarchical community structure and long-range correlations.

With networks showing such a wide-array of properties, it becomes very tedious and challenging to both, model these networks and to study their response to internal failures and external attacks. Given this complexity in the networks, conducting a systematic analysis of their responses requires us to isolate the effects of loss of links on individual properties. Alternatively this also provides insight into how individual properties affect each other and the overall robustness of the network. In order to do this, we require ensembles of networks that dominantly exhibit only the relevant properties while