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New insights from the canonical fisheries model – Optimal management when stocks are low

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ABSTRACT

We analyse the standard optimal control fishery biomass model and derive some novel results on optimal management when fish stocks are low. We show that as long as it is not optimal to let the stock become extinct and the marginal benefit of harvesting is bounded below infinity for all harvest levels, there will always be an interval with low stock sizes where it is optimal not to harvest. This result does not depend on any assumption that marginal harvesting cost per unit increases with decreasing stock size. We then prove that under weak conditions the shadow price on the fish stock always goes to infinity as the stock approaches zero. The results are generalized to a particular class of age structured models.

1. Introduction

Clark (1973) and Clark and Munro (1975) presented dynamic fishery models that gave the theory of renewable resources a proper capital theoretic foundation. The basic fishery model entails one control variable, one state variable; the planning horizon is infinite time and the problem is autonomous. When the profit function is nonlinear in the control variable and there is an optimal path to the steady state, this steady state should be approached gradually along two saddle paths, or stable manifolds (see, e.g., Kamien and Schwarz, 1991). The standard model has usually applied an ecological lumped parameter model of the form $\dot{x} = G(x) - h$ where x is the size of the fish stock in biomass and h is the harvest rate. It has been recognized for a long time that optimal extinction in these models depends on the relative magnitude of the interest rate and the intrinsic growth rate, $G'(0)$, in addition to the unit cost of harvesting (Clark, 1973; Cropper et al., 1979). Although this model is well understood, some wrinkles remain to be ironed out. One is the question of harvest levels at low stock levels, where it is has been known that in some versions of the standard fisheries model it is optimal to set harvest equal to zero for low stock levels. This is commonly attributed to either the bang-bang nature of problems that are linear in the control, Clark and Munro (1975) or an assumption that harvest costs are stock dependent and that the marginal cost of harvest becomes infinite when the stock approaches zero, (Leung and Wang, 1976; Lewis and Schmalensee, 1977).[1]

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[1] On the other hand, if the marginal benefit of harvesting goes to infinity as the harvest rate goes to zero, typically in models with some type of iso-elastic instantaneous utility, then if stocks are strictly positive it is always optimal with some strictly positive harvest rate, Levhari and Mirman (1980). Whether extinguishing the fish stock is optimal will also in this case depend on the relationship between the discount rate and the intrinsic growth rate.

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In what follows, we show that these assumptions are not necessary. In order to properly analyse optimal harvest levels at low stocks, it is crucial to examine the behaviour of the shadow price at low stock levels. We argue below that analysing the properties of the shadow price is equivalent to analysing the stable saddle path in a phase diagram in stock/shadow price space. If we interpret the stable saddle path as a function that maps the state variable into the shadow price it is evident that the stable saddle path is in fact the derivative of the value function. We then demonstrate that the shadow price of a renewable resource goes to infinity if the growth in the resource is zero at zero stock. This fact has remarkably not been noted in the literature, except for the case where revenue is a linear function of harvest levels, Nævdal (2016). In his milestone book on natural resource economics Colin Clark stays silent on this. He draws the basic fishery model phase-diagram in the stock – harvest space, but the saddle path is not drawn for low harvest levels, Clark (1991, p. 99) and also Conrad and Clark (1987, p. 56). In the well-recognized book by Leonard and Long (1992) on optimization and dynamic control models, the saddle path illustrating a schooling fishery is only indicated for a restricted set of values in the stock – shadow price space (Leonard and Long (1992, p. 296) and is not drawn for values of the stock close to zero.

In Section 2 below, we first formulate and analyse our baseline model exemplified by a schooling fishery where the net harvest benefit is a concave function of harvest. In Section 3, we next apply fast/slow-dynamics and show that the results apply to at least some age structured models. Section 4 concludes the paper with a discussion of the results and relating them to the concept of harvest control rules.

2. The canonical fisheries model

The following is the basic version of the fisheries model where a schooling fishery is considered. In a schooling fishery there are no stock dependent harvest costs. We assume that the net instantaneous benefits from harvesting is given by a continuous and strictly concave function $D(h)$ with $D(0) = 0$, and where $D'(h) > 0$ over an interval $[0, h_{max}]$ where $h_{\max} \leq \infty$. For notational convenience we denote $D'(h)$ as $d(h)$. Note that strict concavity of $D(h)$ ensures that $d(h)$ has an inverse defined for all positive values of its argument. In order to ensure that our results are not the result of assuming infinite derivatives of $D(h)$, we postulate that $0 < d(0) < \infty$ which is a crucial assumption driving our results. The natural growth function $G(x)$ is taken to be strictly concave and satisfy $G(0) = 0$, $G'(x) > 0$ over some interval $[0, \bar{x}\,)$ and $G'(x) < 0$ for $x > \bar{x}$. We assume that the intrinsic growth rate exceeds that of the discount rate, $G'(0) > \rho$, which is reasonable for most fish species It is also assumed that there is some number $K > \bar{x}$, denoted carrying capacity, such that $G(K) = 0$. The specification of $G(x)$ is in line with standard growth functions such as the logistic one, which is used in our numerical illustrations. The assumptions lead to the following optimization problem:

$$V(x(0)) = \max_{h \geq 0} \int_0^\infty D(h) e^{-\rho t} dt \quad \text{subject to } \dot{x} = G(x) - h, \text{ and } x(0) \text{ given,} \tag{1}$$

where $h \geq 0$ is the harvest and $x \geq 0$ is the size of the fish stock and $\rho \geq 0$ is the discount rate. The current value Hamiltonian for this problem is:

$$H = D(h) + \mu(G(x) - h). \tag{2}$$

Here μ is the co-state variable. The Hamiltonian is concave in (h, x), so sufficiency theorems such as Theorem 9.11.1 in Sydsæter et al. (2005) are fulfilled. The necessary conditions become:

$$\frac{\partial H}{\partial h} = d(h) - \mu \leq 0 \; (= 0 \text{ if } h > 0) \tag{3}$$

and

$$\dot{\mu} = (\rho - G'(x))\mu. \tag{4}$$

(3) follows from maximising the Hamiltonian with respect to h, when H is a concave function of h. Transversality conditions must also be checked. By assumption there exist a steady state and we show below that the optimal path converges to this steady state from any $x(0) > 0$. It is straightforward to check that $\lim_{t \to \infty} \mu(t)(y(t) - x(t))e^{-\rho t} \geq 0$ where $x(t)$ is the optimal state variable and $y(t)$ all other admissible functions. As $y(t)$, $x(t)$ and $\mu(t)$ are all finite, this expression goes to zero. The transversality condition given in Theorem 9.11.1 in Sydsæter et al. (2005) therefore holds and with the rest of our assumptions implies that sufficient conditions for optimality hold. Control condition (3) implies that $d\,(0) < \mu \Rightarrow h = 0$ and condition (3) may be rewritten as:

$$h = \max\left(0, d^{-1}(\mu)\right). \tag{5}$$

Inserting Eq. (5) into the natural growth equation yields next: