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Optical solitons in birefringent fibers with quadratic–cubic nonlinearity by extended trial function scheme

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ABSTRACT

This paper employs extended trial function scheme to derive soliton solutions in birefringent fibers with quadratic–cubic nonlinearity. The mathematical algorithm reveals bright and singular optical soliton solutions that are listed with their respective existence criteria.

1. Introduction

The study of optical solitons with quadratic–cubic (QC) nonlinearity is going on for more than a couple of decades. There are several results with a variety of mathematical methods that are reported. These include traveling wave hypothesis, method of undetermined coefficients, semi-inverse variational principle, conservation laws and various other aspects [1–10]. This law of nonlinearity first appeared in 1994 followed by its re-appearance during 2011 and ever since it has gained popularity and several results have flooded across a variety of journals [9,10]. While all of these results are for polarization-preserving fibers, it is now time to move on to the next chapter, after turning the page. This paper will be studying the QC nonlinearity with polarization-mode dispersion (PMD). The corresponding governing equations are listed for birefringent fibers and QC nonlinearity with four-wave mixing (4WM) effect included. They are subsequently addressed by extended trial function method to retrieve bright and singular optical solitons. The details are explored in the rest of the paper after a quick introduction to the governing model.

2. Governing model

The NLSE with QC nonlinearity for polarization-preserving fibers is written as [5]

$$iq_t + aq_{xx} + (b_1\,|q| + b_2\,|q|^2)q = 0. \tag{1}$$

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Upon splitting this equations for birefringent fibers into two components with four-wave mixing, we arrive at:

$$iu_t + a_1u_{xx} + b_1u\sqrt{|u|^2 + |v|^2 + uv^* + u^*v} + (c_1\,|u|^2 + d_1\,|v|^2)u + p_1v^2u^* = 0, \tag{2}$$

$$iv_t + a_2v_{xx} + b_2v\sqrt{|v|^2 + |u|^2 + u^*v + uv^*} + (c_2\,|v|^2 + d_2\,|u|^2)v + p_2u^2v^* = 0. \tag{3}$$

Eqs. (2) and (3) is the governing model for soliton transmission through birefringent fibers with 4WM.

3. Mathematical preliminaries

The starting hypothesis for solving the considered coupled system is given by

$$u(x,\,t) = P_1[\zeta(x,\,t)]\exp[i\phi(x,\,t)], \tag{4}$$

$$v(x,\,t) = P_2[\zeta(x,\,t)]\exp[i\phi(x,\,t)], \tag{5}$$

where $P_l(\zeta)$ for $l = 1, 2$ represent the amplitude component of the soliton and ϕ is the phase component of the soliton that is described as

$$\phi = -\kappa x + \omega t + \theta, \tag{6}$$

and also

$$\zeta = x - \nu t. \tag{7}$$

Here, ν is the velocity of the soliton, κ is the frequency of the solitons in each of the two components while ω is the soliton wave number and θ is the phase constant. Putting (4) and (5) into (2) and (3), and decomposing into real and imaginary parts respectively yield

$$-(\omega + a_l\kappa^2)P_l + b_lP_l^2 + b_lP_lP_{\bar{l}} + c_lP_l^3 + (d_l + p_l)P_lP_{\bar{l}}^2 + a_lP_l'' = 0, \tag{8}$$

$$(\nu + 2a_l\kappa)P_l' = 0, \tag{9}$$

for $l = 1, 2$ and $\bar{l} = 3 - l$. From Eq. (9), one can obtain the speed of the soliton as

$$\nu = -2a_l\kappa. \tag{10}$$

Equating the two values of the soliton velocity (10) leads to

$$a_1 = a_2. \tag{11}$$

Therefore, it makes sense to define

$$a_1 = a_2 = a, \tag{12}$$

and then speed of the solitons for both components reduce to

$$\nu = -2a\kappa. \tag{13}$$

Therefore, the coupled system (2) and (3) change to

$$iu_t + au_{xx} + b_1u\sqrt{|u|^2 + |v|^2 + uv^* + u^*v} + (c_1\,|u|^2 + d_1\,|v|^2)u + p_1v^2u^* = 0, \tag{14}$$

$$iv_t + av_{xx} + b_2v\sqrt{|v|^2 + |u|^2 + u^*v + uv^*} + (c_2\,|v|^2 + d_2\,|u|^2)v + p_2u^2v^* = 0. \tag{15}$$

Thus, the real part equations (8) can be designed as

$$-(\omega + a\kappa^2)P_l + b_lP_l^2 + b_lP_lP_{\bar{l}} + c_lP_l^3 + (d_l + p_l)P_lP_{\bar{l}}^2 + aP_l'' = 0. \tag{16}$$

Next, employing the balancing principle, one has a relationship as

$$P_{\bar{l}} = P_l. \tag{17}$$

Consequently, Eq. (16) modifies to

$$-(\omega + a\kappa^2)P_l + 2b_lP_l^2 + (c_l + d_l + p_l)P_l^3 + aP_l'' = 0. \tag{18}$$

4. Extended trial function method

This section will focus on the integration of (2) and (3) by the aid of a mathematical tool called extended trial function scheme [6–8]. For soliton solutions to (18), the following assumption is taken to be

$$P_l = \sum_{i=0}^{\varsigma} \varrho_i^{(l)}\varpi^i, \tag{19}$$