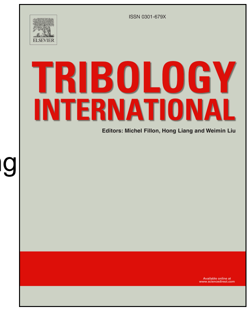

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PII: S0301-679X(18)30450-X
DOI: 10.1016/j.triboint.2018.09.011
Reference: JTRI 5399

To appear in: *Tribology International*

Received Date: 27 June 2018
Revised Date: 3 September 2018
Accepted Date: 12 September 2018

Dynamic Coefficients and Stability Analysis of Finite-length Journal Bearings Considering Approximate Analytical Solutions of the Reynolds Equation

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Abstract

The paper presents linearized coefficients for finite-length journal bearings based on the HD forces with multiplicative correction polynomials applied to the IS and the IL solutions under the assumption of π-film boundary condition. Based on the linear theory, stability thresholds and their dependence on the length-to-diameter ratio are investigated. Moreover, using the numerical continuation method the change of the generalized Hopf bifurcation, which separates regions with subcritical and supercritical Hopf, is documented. All results are compared with those of the solution obtained using the finite differences method.

Keywords: Reynolds equation, Analytical solution, Dynamic coefficients, Stability analysis

1. Introduction

Journal bearings are important in the field of rotating machinery because they can provide a support of high-speed rotating parts or of high radial loads. In such applications, there is some viscous lubricant between a rotating *shaft* and a non-rotating bearing *shell*. Stable trajectories of the journal are located close to its static equilibrium position, which is determined by the weight of the supported rotor and by external loads. The static equilibrium position generally does not coincide with the geometrical center of the bearing. Misalignment and the relative motion of journal and the shell surfaces are essential conditions for the occurrence of hydrodynamic pressure in the lubricant. The concept of the separation of the journal's and shell's surfaces by a thin viscous film with the load carrying ability is known as *hydrodynamic lubrication* [40].

Fundamentals of hydrodynamic lubrication were formulated by Reynolds, who derived a partial differential equation governing the pressure distribution of thin viscous fluid films in 1886 [35]. An analytical closed form solution of the *Reynolds equation* has been found lately [37]. However, the most used approach to solving the Reynolds equation is the usage of numerical methods — most notably the finite difference method (FDM) [40], the finite element method (FEM) [23] and the finite volumes method (FVM) [34]. Alternatively, approximate analytical solutions can be used. For *long bearings* with length-to-diameter ratio $\eta > 2$ the infinitely long journal bearing (ILJB) approximation [39] can be employed, and for *short bearings* with $\eta < 0.5$ the infinitely short journal bearing (ISJB) approximation can be employed [32]. Nevertheless, these approximative solutions do not hold for *finite-length* or simply *finite journal bearings* (FLJB) with length-to-diameter ratios from in range $0.5 \leq \eta \leq 2$.

If the hydrodynamic pressure is integrated over the bearing surface one obtains a *hydrodynamic force*. The hydrodynamic pressure is a function of the time, the journal position and its speed, and therefore the hydrodynamic force is non-linear. This force can be adopted in equations of motion of the rotor [30] but it is far more common — especially in the case of stationary rotating machinery — to replace the non-linear hydrodynamic force by so-called *dynamic coefficients*.

History of the dynamic coefficients goes back to the year 1925, when Stodola realised that the elastic part of the hydrodynamic forces is almost linear, if operating conditions of the journal are steady [41]. Okazaki and Hori were studying stability of the journal bearings 30 years later and used a linearised analytical model of the hydrodynamic force which consists of 4 elastic and 4 damping linear coefficients defined in radial and tangetial directions [33]. Holmes adopted a similar model formulated in the Cartesian coordinate system [19]. Lund and Sternlicht showed that the dynamic coefficients can be evaluated numerically as the derivative of the hydrodynamic force with respect to the displacement and the velocity of the journal [26]. Furthermore, they defined a *cross-coupling* term. The cross-coupling occurs in many types of bearings and can be observed, if the bearing is loaded vertically. Then the journal moves not only vertically but also horizontally.

The dynamic coefficients as well as other parameters of the finite journal bearings may be estimated employing the ISJB and the ILJB approximations. However, such a practice should be treated with caution as the above-mentioned approximations may overpredict or underpredict the parameters of the finite bearing and are more suited for the evaluation of upper or lower limits or for determination of trends [43]. Some authors utilized the ISJB and the ILJB approximations as basic closed-form solutions, which were later corrected with corrective factors in order to be applicable to the FLJBs. One of the first attempts to correct the ILJB approximation was proposed by Fedor in

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