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On mechanical damping of cantilever beam-based electromagnetic resonators

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ABSTRACT

Often when optimising a vibration energy harvester, the mechanical damping is given little significance and is usually assumed to be a constant. This paper analyses the importance of mechanical damping variation in modelling the behaviour of a cantilever beam-based electromagnetic resonator. It is shown that for beam volumes above 100 mm^3, material damping dominates thermoelastic and air damping, hence becoming the major contributor towards the mechanical damping. A novel method is proposed to define material damping in terms of the maximum critically damped stress at resonance. The new method is shown to be simpler and more accurate than previous methods. Using the developed governing equations, the conditions of optimum load resistance are derived for two particular cases. A comparison is made between the mechanical damping model and the constant mechanical damping assumption in terms of maximum power output. Different trends were noted between the two compared methods, suggesting that the constant mechanical damping assumption can lead to large errors in power prediction. Further analysis describes the existence of an optimum mass ratio for electromagnetic resonators operating under a low magnetisation parameter. Lastly, this paper shows that different frequency tuning methods are preferable under different operating conditions.

1. Introduction

In the mid-90s, William and Yates [1] proposed the concept of vibration energy harvesting as a viable source of sustainable energy to power small electronics. Research in this area has then drastically been increased over the past decade. There exist several methods to convert mechanical vibrations into electrical power, with the two most common methods being electromagnetic induction and piezoelectric conversion. Piezoelectric conversion method is usually the preferred option due to its high power output at small volumes [2]. However, electromagnetic induction method was shown to be better than piezoelectric for device volume higher than 500 mm^3 [3]. Beeby et al. [4] demonstrated that if a detailed optimisation was perform, the power output of an electromagnetic resonator can surpass a piezoelectric resonator at a volume of 100 mm^3.

The dynamics of a vibrating structure is usually characterised based on its natural frequency and damping capacity. While thorough studies and numerous analytical methods have been conducted on the natural frequency of a structure [5,6], little is known about damping despite its significance. For cases of cantilever beam-based electromagnetic resonators, there are two sources of damping which are the mechanical damping of the beam and the electromagnetic damping due to induced

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magnetic field [7]. Normally, the mechanical damping of the beam is obtained experimentally as no analytical methods exist to predict this damping. However, the electromagnetic damping can be estimated analytically [8]. Application wise, cantilever beam-based resonators are usually optimised experimentally [9] or by assuming a constant mechanical damping in an analytical optimisation algorithm [10,11]. Currently, the most common method to determine the mechanical damping of a structure is the logarithmic decrement method and the half-power bandwidth method [12]. However, these methods are purely experimental.

The mechanical damping in a common cantilever beam-based resonator design is generally the sum of three damping components which are the material damping, air damping and thermoelastic damping [13]. Although other components of mechanical damping exist, the three mentioned components are the most common source of mechanical damping for cantilever beam-based applications. Material damping arises from the atomic arrangement of the beams as well as its impurities. Hence, material damping is a type of microscopic damping as it exists at a molecular level. Due to this, different materials have different damping properties. In an earlier study, Lazan [14] proposed in his work that a strong relationship exists between the stress experienced in a vibrating structure and the energy dissipated by the structure (damping energy). His study focused on material damping and he showed that different damping-stress curves exist for different materials. With this, Lazan developed a generalised damping-stress equation for most metals, relating the loss factor of a metallic material to its maximum bending stress and its fatigue limit stress. Kume et al. [15] refined Lazan's damping-stress equation by considering the stress contours in a cantilever and derived a new damping-stress relation, although the results of their equation are very similar to Lazan's. Gounaris and Anifantis's [16] applied the same approach in finite element analysis to analytically determine the material damping of a beam-like structure. Nevertheless, all of these methods are related to the same damping-stress equation and may not be entirely accurate due to the generalisation of the equation [17]. In addition, there were no evidences that the methods presented were valid for beams with tip mass applications. Edberg [18] showed that for small damping ratios, the product of the beam eigenvalue squared and the damping ratio is a constant. However, this theory did not take into account the effects of the forcing input applied to vibrate the beam. To this day, there has been no exact analytical approach to predict the material damping of cantilever beams.

Zener [19] described thermoelastic damping as a form of the energy losses due to the temperature gradient generated by the repeated state of tension and compression in a vibrating beam. Several studies have then recorded the significance of this damping as the main contributor to the mechanical damping in micro-sized structures [20–22]. However, Alblas [23] deemed this form of damping to be negligible in macro-sized structures. Many authors have researched on the importance of ambient air damping due to air drag on the vibration of a cantilever beam [24–26]. Similar to the thermoelastic damping, this form of damping is more significant in the micro-sized region as compared to the macro-size region. Nevertheless, an analytical approach exists to predict both the thermoelastic and air damping of cantilever beam structures. Iourtchenko et al. [27] took the approach to identify the overall damping of a vibrating structure based on the velocity of the structure at resonance. The method considers a single degree of freedom (SDOF) model where the damping parameter is assumed to be an unknown odd function of the velocity at resonance. By applying Schlomilch's integral to the steady-state solution for a slowly varying amplitude and phase, an analytical equation for the unknown damping can be derived. However, velocity if not a unique parameter of damping. This means that if a damping-velocity relation was formed for a cantilever beam with a specific tip mass, it would not be applicable for other tip masses although the same beam properties and size were used, due to the inertial effect of the different tip mass. Nevertheless, the damping-velocity relation of a material for a specific tip mass (or even no tip mass) can be applied for any beam size, suggesting that this relation is only a function of the tip mass inertial term.

This study focused on the analysis of mechanical damping for macro-sized cantilever beam-based electromagnetic resonators as significant applications of this resonators lies in the macro range (>100 mm^3). The equations of motion for cantilever beams with tip mass were derived based on the Euler-Bernoulli beam theory as the geometry of beams used in resonators are usually slender enough to allow neglecting the effects of shear deformation. An analysis was then performed to determine the significance of each damping component of the mechanical damping in macro scale analysis. A method was then proposed to predict the unknown material damping of cantilever beams base on the hysteretic damping model. A comparison was made between the power output of an electromagnetic resonator using the developed mechanical damping model and using the constant mechanical damping assumption. Conditions of certain optimum parameters for two particular cases were derived in this work. Finally, a discussion was made on the preferred method of frequency tuning for electromagnetic resonators.

2. Governing equations

To fully model the behaviour of a cantilever beam-based electromagnetic resonator, the following equations are required:

- Cantilever beam equation of motion
- Electromagnetic power equation
- Damping equations

This section describes the derivation of each governing equation to model the electromagnetic resonator.