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A non-intrusive reduced basis approach for parametrized heat transfer problems

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ABSTRACT

Computation Fluid Dynamics (CFD) simulation has become a routine design tool for i) predicting accurately the thermal performances of electronics set ups and devices such as cooling system and ii) optimizing configurations. Although CFD simulations using discretization methods such as finite volume or finite element can be performed at different scales, from component/board levels to larger system, these classical discretization techniques can prove to be too costly and time consuming, especially in the case of optimization purposes where similar systems, with different design parameters have to be solved sequentially. The design parameters can be of geometric nature or related to the boundary conditions. This motivates our interest on model reduction and particularly on reduced basis methods. As is well documented in the literature, the offline/online implementation of the standard RB method (a Galerkin approach within the reduced basis space) requires to modify the original CFD calculation code, which for a commercial one may be problematic even impossible. For this reason, we have proposed in a previous paper, with an application to a simple scalar convection diffusion problem, an alternative non-intrusive reduced basis approach (NIRB) based on a two-grid finite element discretization. Here also the process is two stages: *offline*, the construction of the reduced basis is performed on a fine mesh; *online* a new configuration is simulated using a coarse mesh. While such a coarse solution, can be computed quickly enough to be used in a rapid decision process, it is generally not accurate enough for practical use. In order to retrieve accuracy, we first project every such coarse solution into the reduced space, and then further improve them via a rectification technique. The purpose of this paper is to generalize the approach to a CFD configuration.

1. Introduction

During the past fifty years electronic devices and systems kept becoming smaller and smaller, this growing need for miniaturization led to an increasing high heat production. To avoid any possible failure or malfunction of electronics devices and ensure their reliability, it is essential to maintain the temperature of the electronic components below an acceptable

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upper limit. Cooling of electronic systems is consequently essential in controlling the component temperature and avoiding any hot spot. Designing cooling systems for miniaturized electronics devices presents difficult challenges to mechanical engineers and analysts. Average while, computational modeling is gaining popularity, particularly Computation Fluid Dynamics (CFD) modeling which has become a routine design tool for predicting accurately thermal performance of electronics cooling system. Although CFD modeling can be used at different scales, from component/board levels to larger system, classical discretization techniques such as finite volume or finite element methods can prove to be limited by memory space and long calculation times, which can be problematic. Inexpensive and accurate computational tools to predict the fluid flow and heat transfer can be very useful, specially when thermal analysis is done at the end of the design process where time constraints are greatest, hence our focus on model reduction and particularly to reduced basis methods (see [9,14,17,19]).

Reduced basis method exploits the parametric structure of the governing PDEs to construct rapidly, convergent and computationally efficient approximations. Previous work on the reduced basis method in numerical fluid dynamics has been carried out by [12,16,20] and more particularly for the Navier–Stokes equations [7,18,23,27,28] which requires treatment of non-linearity and non-affine parametric dependence. More recent works with turbulent flows can be found in [2,26]. Let σ be a set of parameters associated to our physical system, these methods rely on the fact that when the parameters vary, the manifold of solutions is often of small (Kolmogorov) dimension. In this instance, there exists a set of N particular values of σ taken in $\mathcal{D}$ (the parameter space) from which one can build a basis. This basis, called reduced basis, is made of the solutions $u(\sigma_1), \cdots, u(\sigma_N)$ and can approach any solution $u(\sigma)$, $\sigma \in \mathcal{D}$. Thus, when the σ_i are well chosen,[1] the size of the reduced basis is much smaller compared to the number of degrees of freedom of the problem discretized by a classical method (finite element, finite volume, or other). The standard reduced basis method is a Galerkin approach within the reduced basis approximation space, thus the reduced basis approximation of the "truth" solution is obtained by the resolution of a small dimensional linear system.

One of the keys of this technique is the decomposition of the computational work into *offline/online* stages. However, the decomposition of the matrices into *offline/online* pieces requires modifying the calculation code, leading to an intrusive procedure. Examples of the standard reduced basis method applied to heat transfer problems can be found in [8,21,24,25]. In some situation — with a commercial CFD code for example – it is not possible to perform all the *offline* computations required to have a inexpensive and fast *online* stage. For this reason, we proposed to use an less intrusive reduced basis method, introduced in [3,4], where coarse triangulations are used to compute coarse approximation during the *online* stage. Recently, other non-intrusive Reduced Order Methods (ROM) for fluid dynamics have been proposed [5,29], those ROM are based on proper orthogonal decomposition and Radial Basis function (RBF) to compute the coefficients of the reduced model. During the online computation, for any given (untrained) parameter an interpolation approach using RBF as interpolation functions is used to estimate the coefficients of the POD decomposition. As in our method we use coarse triangulations to compute coefficients of the RB decomposition and then further improve them via a rectification technique, keeping a physical meaning to the approach.

The aim of this paper is to provide tests to validate and generalize our method for heat transfer problems. In Section 2, we provide a brief introduction to reduced basis methods and the methodology of the non-intrusive reduced basis method. In Section 3, we give a brief description of simple models of cooling devices; we formulate the physical system, the governing equations and boundary conditions. In Section 4, we discuss our numerical experimentations and present the results and conclusions.

2. Methodology

Let us consider the nonlinear parametrized PDEs describing our physical system, over a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3. In these governing equations, θ represent the temperature, u_i the components of the velocity vector field $\mathbf{u}$ in the x_i-directions, p the pressure and σ is a set of n_p parameters related to physical properties or boundary conditions. We denote by $\mathcal{D} \subset \mathbb{R}^{n_p}$ the set of parameters.

We now introduce the variational formulation of our parametrized PDEs :

$$\begin{cases} \text{for a given } \sigma \in \mathcal{D}, \text{find } (\theta, \mathbf{u}, p) \equiv (\theta(\sigma), \mathbf{u}(\sigma), p(\sigma)) \in X \times Y \times Q \text{ such that} \\ \mathcal{F}(\theta, \mathbf{u}, p\,; T, \mathbf{v}, q;\, \sigma) = (0, \mathbf{0}, 0)^t, \qquad \forall (T, \mathbf{v}, q) \in X \times Y \times Q\,, \end{cases} \tag{1}$$

where $\mathcal{F}$ is a functional with the appropriate properties, X, Y and Q are appropriate functional spaces. Let $\{\mathcal{T}_h\}_h$ be a family of regular triangulation of Ω, we denote by X_h, Y_h and Q_h finite element approximation subspaces of respectively X, Y and Q over $\mathcal{T}_h$. The discrete velocity space Y_h and the discrete pressure space Q_h are chosen in order to satisfy the inf-sup condition [6,13,22].

The finite element discretization of (1) is as follows:

$$\begin{cases} \text{for a given } \sigma \in \mathcal{D}, \text{find } (\theta_h, \mathbf{u}_h, p_h) \equiv (\theta_h(\sigma), \mathbf{u}_h(\sigma), p_h(\sigma)) \in X_h \times Y_h \times Q_h \text{ such that} \\ \mathcal{F}(\theta_h, \mathbf{u}_h, p_h\,; T, \mathbf{v}, q;\, \sigma) = (0, \mathbf{0}, 0)^t, \qquad \forall (T, \mathbf{v}, q) \in X_h \times Y_h \times Q_h. \end{cases} \tag{2}$$

[1] A classical and efficient approach to choose the σ_i is the greedy method [1,11].