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Virtual tetrahedral gap element to connect three-dimensional non-coincident interfaces

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ABSTRACT

This study introduces a new version of a virtual tetrahedral gap element to connect partitioned structures which are independently discretized with tetrahedral elements. Tetrahedral meshes are widely used for practical engineering problems due to their simplicity. The proposed interface method employs the localized Lagrange multiplier method. The virtual tetrahedral gap elements are placed between the frame-slave and frame-master interfaces. The surface of the tetrahedral meshes is triangular; thus, a virtual tetrahedral gap element is developed. A distinct feature of the virtual tetrahedral gap element is that it has a zero-strain condition which provides the exact interface reaction forces at the non-matched interface. The proposed tetrahedral gap element handles three-dimensional interface problems more effectively than conventional segment-to-segment methods. It also provides better accuracy. The validity and robustness of the proposed method are demonstrated by several numerical examples.

1. Introduction

Interface schemes include domain decomposition, fluid-structure interaction, and contact and crack analyses [1–4]. These analyses generally use triangular elements in two-dimensional(2D) problems and tetrahedral elements in three-dimensional(3D) problems because it is easy to perform the analyses. The main purpose of the interface analysis is to connect structures that contain different meshes at common boundaries to ensure continuity in a consistent manner. This study considers mesh-tying constraints for independent tetrahedral meshes of a partitioned structure. An important step in the finite element method is to create a discretization model. When performing a 3D engineering analysis, the tetrahedral element is the preferred mesh to create a finite element model from the computer aided design(CAD) model [13–15]. In addition, tetrahedral meshes are more convenient than hexahedral meshes. Based on these properties, tetrahedral elements are used for various engineering interface problems, such as bulk-forming analysis with large deformation. Moreover, remeshing is essential for large deformation. Using the tetrahedron in the remeshing procedure makes the mesh adaptation easier. In fluid-structure interface problems, tetrahedral elements provide a better shape representation of structures, such as blades with complex shapes. In the field of biomechanics, virtual surgery uses deformation and contact analysis of nonlinear materials based on the finite element method. The expression of the exact shape of the body part is essential for making accurate medical decisions. To this end, tetrahedral elements are actively used, and the interface analysis for tetrahedral elements is required. Substructures have different sizes of meshes; hence, the interface nodes of each substructure do not match along the shared boundaries. The mortar method, which imposes the interface constraint in a weak sense, is first introduced for the domain decomposition. The mortar method is more robust than the single pass method; thus, it is widely used in interface methods.[20,30-35] Various domain decomposition studies have been performed based on the mortar method to connect the spatial grids. Contact constraints in the variational equations can be applied in several ways. The method of Lagrange multipliers is widely used in the mortar method [5,6,10–12,27,28]. The Lagrangian multipliers act like contact forces between the substructures. The Lagrangian multipliers are approximated by shape functions and impose contact constraints on the corresponding interpolated displacements to optimally satisfy contact constraints. In the work of Dohrmann et al. [22,36], mesh tying problems were addressed for the 3D dissimilar interface. However, a major concern of the interface method for 3D surfaces is to accurately and efficiently integrate mortar constraints to conserve momentum and energy [16–19].

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Song et al. [25,26] recently introduced the virtual gap element approach based on the localized Lagrange multiplier for the 2D interface problem and 3D interface problem with the brick element. The gap element method provides better accuracy and efficiency for these cases. From there, we have shown the possibility to extend the boundaries of the research on the tetrahedral element, but there is no detail and corresponding analysis. This paper describes in detail the tetrahedral gap elements that include the discrete frame construction method using Delauney tessellation.

Brick elements provide better finite element analysis results than tetrahedral elements. In addition, tetrahedral elements exhibit a behavior that is too stiff as well as volumetric locking problems. Nevertheless, tetrahedral meshes are still preferred for complex engineering problems. Numerous approaches have been proposed to overcome the disadvantages of the analysis that employs tetrahedral elements [21–23]. Tetrahedral elements have recently been employed in the interface area, and hexahedral elements are used in the interior of the structure to satisfy both geometric representation and accurate numerical analysis. Therefore, it is important to have a proper scheme to deal with tetrahedral elements in the interface analysis.

The remainder of this paper is organized as follows: Section 2 briefly reviews the variational formulation with interface constraints and the mortar integration scheme. Section 3 describes the contact frame construction with triangular meshes. The present method is based on the localized Lagrange multiplier method; hence, it exploits frame structures with triangular meshes. Section 4 introduces a virtual tetrahedral gap element, which is a key element of the present work. Section 5 provides numerical examples. Section 6 gives the concluding remarks.

2. Variational formulation with interface constraints

The mesh-tying problems of two elastic material bodies are considered throughout this paper. Two substructures are marked with superscripts 1 and 2 for the sake of illustration. Typically, 1 refers to the master, and 2 is the slave.

2.1. Interface virtual work

Two divided bodies are designated as Ω^1 and Ω^2. The body surfaces are represented as Γ^1 and Γ^2. The boundary covers all specific boundary conditions, and does not overlap as follows:

$$\Gamma_u^a \cup \Gamma_\sigma^a \cup \Gamma_C^a = 0 \tag{1}$$

$$\Gamma_u^a \cap \Gamma_\sigma^a = \Gamma_C^a \cap \Gamma_\sigma^a = \Gamma_u^a \cap \Gamma_C^a = \phi, \quad a = 1,\ 2$$

where Γ_u is the Dirichlet boundary with given displacements, and Γ_σ is the Neumann boundary with prescribed tractions. Γ_C is a common interface. The subdomains must satisfy Eq. (1). In Fig. 1, the mesh-tying constraint is represented as follows:

$$\mathbf{u}^1(\mathbf{x}^1) - \mathbf{u}^2(\widehat{\mathbf{x}}^2) = 0 \; on \; \Gamma_C \tag{2}$$

The above equation indicates that the relative displacement of the substructures is zero. In other words, the divided bodies are joined together. No distinction is found between the tangential and normal directions of the displacement vector. The total potential energy consists of the strain energy of the system, Π^{int}, the potential energy of external work, Π^{ext}, and the contact work with the Lagrange multiplier, Π_C^{LM}. For elastic materials, these terms are expressed as:

$$\Pi = \sum_{i=1}^{2} \Pi^{int(i)} + \sum_{i=1}^{2} \Pi^{ext(i)} + \Pi_C^{LM}, \tag{3a}$$

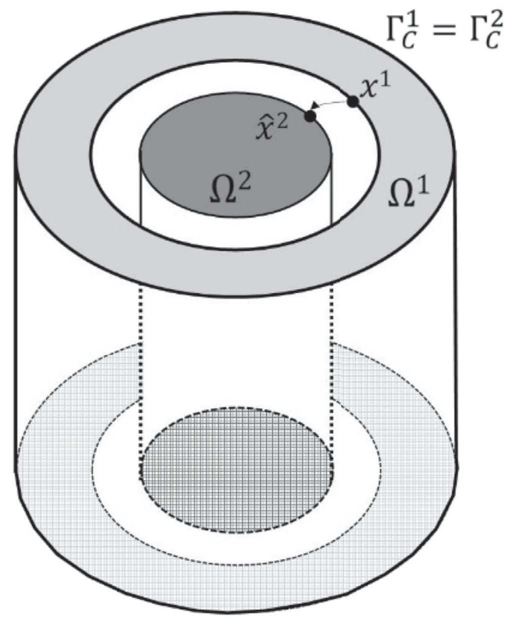

Fig. 1. Interface constraint.

$$\Pi^{int} = \frac{1}{2}\int_\Omega \epsilon^T \mathbf{D} \epsilon d\Omega, \tag{3b}$$

$$\Pi^{ext} = -\int_\Omega \mathbf{u}^T \mathbf{b} d\Omega - \int_\Gamma \mathbf{u}^T \bar{\mathbf{t}} d\Gamma, \tag{3c}$$

$$\Pi_C^{LM} = \int_\Gamma \boldsymbol{\lambda}(\mathbf{u}^1 - \mathbf{u}^2) d\Gamma. \tag{3d}$$

where ϵ is the strain; $\mathbf{D}$ is the elastic material moduli; and $\boldsymbol{\lambda}$ represents the Lagrange multiplier. In addition, $\mathbf{b}$ and $\bar{\mathbf{t}}$ are the body force and surface traction, respectively. The principle of the virtual work can be expressed as:

$$\begin{aligned} \delta\Pi &= \sum_{i=1}^{2} \delta\Pi^{int(i)} + \sum_{i=1}^{2} \delta\Pi^{ext(i)} + \delta\Pi_C^{LM} = 0 \\ &= \sum_{i=1}^{2}\left(\int_\Omega \delta\epsilon^T \mathbf{D} \delta\epsilon d\Omega\right) + \sum_{i=1}^{2}\left(-\int_\Omega \delta\mathbf{u}^T \mathbf{b} d\Omega - \int_\Gamma \delta\mathbf{u}^T \bar{\mathbf{t}} d\Gamma\right) + \int_\Gamma \boldsymbol{\lambda}(\delta\mathbf{u}^1 - \delta\mathbf{u}^2) d\Gamma \end{aligned} \tag{4}$$

The mortar method based on the segment-to-segment approach is implemented to the last term of the above equation, which is the virtual contact work. Slave nodes are mapped to the master to integrate the virtual contact work. The integration scheme for the interface constraint will be described in the following section.

2.2. Mortar discretization

The displacements and Lagrange multipliers interpolation are given as:

$$\boldsymbol{\lambda} = \sum_{i=1}^{n_\lambda} \boldsymbol{\phi}_i \boldsymbol{\lambda}_i, \tag{5a}$$

$$\mathrm{u}^a = \sum_{i=1}^{n} \mathbf{N}_i^a \mathbf{u}_i^a, \quad a = 1, 2. \tag{5b}$$

The shape function $\mathbf{N}$ is defined with respect to the finite element, and three linear shape functions for the first-order tetrahedral element are