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A linear complete extended finite element method for dynamic fracture simulation with non-nodal enrichments

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ABSTRACT

A linear complete extended finite element method for arbitrary dynamic crack is presented. In this method, strong and weak discontinuities are assigned to a set of non-nodal points on the interface, whereby the discontinuous functions across the interface are reproduced by extended interpolation. The enrichments are described to reproduce both the constants and linear functions on sides of the interface, which are critical for finite element convergence. A key feature of this method is that the enrichment descriptions and the finite element mesh are optimally uncoupled; the element nodes are not enriched facilitating the treatment of crack modeling in object-oriented programs. The enrichment variables are *physically-based* quantities which lead to a strong imposition of both the Dirichlet boundary conditions and the interface conditions. The convergence of the method is validated through static simulations from linear elastic fracture mechanics. The efficacy of the method for modeling dynamic crack propagation is demonstrated through two benchmark problems.

1. Introduction

The extended finite element method (XFEM) [1,2] exploits a local partition of unity [3] to enhance the approximation space by non-polynomial bases. One advantage of the XFEM over the finite element method (FEM) is that it can model the discontinuities in the solution to a given PDE in a local domain without remeshing. However, in a standard XFEM, finite element mesh is locally incorporated in the description of enrichments, i.e., the finite element nodes which belong to mesh, but not to the interface, are enriched to describe a discontinuity across an interface. This will arise many difficulties in XFEM reproducing capabilities, or even the programming implementation [4].

Some remedies have been provided in early XFEM studies to address the difficulties arisen due to dependency between the mesh and the enrichments. Examples include shifting the approximation to hold the Kronecker-δ property introduced by Belytschko et al. [5], treating blending elements to correct reproducing conditions addressed by Chessa et al. [6] and Fries [7], and enriching a subset of element nodes to satisfy C^0-continuity conditions between the enriched element and its contiguous elements discussed by Belytschko et al. [8] and Zi et al. [9]. Moreover, Song et al. [10] introduced the phantom node method to model the discontinuities independently of the mesh. This approach was further investigated for shell elements in Refs. [11,12]. As alternative classes of methods, the cracking-particle method [13] and dual-horizon peridynamics [14,15] in meshfree methods have been successfully used in crack modeling. In these approaches, the crack is modeled by splitting particles on sides of the interface whereby the strong discontinuity is captured independent of the mesh.

One of the main difficulties in enriching finite element nodes is the imposition of Dirichlet boundary conditions. Since the interpretation of enrichment variables is difficult, the non-smooth boundary constraints [16] and interface constraints [17,18] are weakly enforced, i.e., they are imposed in the weak form using Lagrange multiplier techniques. Moreover, some treatments introduced for the XFEM cannot be applied together. For instance, as lumped mass matrices are crucial for the efficiency of the explicit time stepping, Menouillard et al. [19] introduced a mass lumping strategy which is applicable to unshifted enrichment functions. To circumvent such difficulties, it is desirable to have a technique that can describe discontinuities across the interfaces independent of finite element mesh.

Another main difficulty in the standard XFEM is the programming implementation. Since the finite element nodes in a local element are

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used to construct the enrichments, the element object and the enrichment object in an object-oriented program (OOP) become dependent. This feature clearly violates the OOP principle of abstraction as the discontinuities in a continuum are independent from the mesh. As a consequence, several difficulties arise in both XFEM implementation and post-processing steps [20,21].

To overcome these difficulties, a non-nodal enrichment technique [22] was introduced to describe the discontinuities across an interface with minimal incorporating of finite element mesh into enrichment descriptions. In this approach, a partition of unity is not constructed, so the element nodes are not enriched. Instead, discontinuities are defined for a set of non-nodal points on the interface to reproduce discontinuous fields across the interface. The enrichments in Ref. [22] reproduce strong discontinuities in displacement fields, which can well reflect the nature of a crack. Furthermore, since the enrichment parameters were selected to be the displacement jumps which are, in fact, *physically-based* variables related to the crack, the interface boundary conditions can be easily treated in the strong form as Dirichlet boundary conditions.

The main objective of this study is to enhance the interpolation in Ref. [22] to reproduce not only the strong discontinuities in displacement fields but also the discontinuities in their first derivatives. To this end, a new set of enrichment bases is introduced to non-nodal points on the interface to reproduce the jumps in the first derivatives. This leads to a completely local enrichment without shifting techniques and a linear complete approximation, i.e., the interpolation can reproduce both rigid body motion and constant strain state on both sides of the interface.

The remainder of the paper is as follows. In section 2, we provide the non-nodal enrichment displacement for a linear complete approximation in one- and two-dimensional problems. Section 3 presents the strong form, the weak form and the discretizations for dynamic analysis. It also introduce a quadrature rule developed for non-nodal enrichment methods. Finally, section 4 provides several numerical studies analyzed by the method for static and dynamic simulations.

2. Enriched displacement fields for discontinuity

We consider a two-dimensional body Ω with its boundary Γ in the initial configuration, as shown in Fig. 1. The body includes a crack with a surface discontinuity denoted by Γ_c. Two sides of this discontinuity are signed by a continuous level set function $f(\mathbf{X})$ so that $f(\mathbf{X})=0$ gives the discontinuity surface. The level set function f can be described by the signed distance function as

$$f(\mathbf{X}) = \min_{\overline{\mathbf{X}}\in\Gamma_c} \|\mathbf{X}-\overline{\mathbf{X}}\| \ \operatorname{sign}(\mathbf{n}^+\cdot(\overline{\mathbf{X}}-\mathbf{X})) \tag{1}$$

where $\overline{\mathbf{X}}$ is the closest point on the interface to $\mathbf{X}$ and $\|\cdot\|$ denotes the Euclidean norm. The unit normal vector $\mathbf{n}^+$ is perpendicular to the discontinuity surface where the level set is positive, i.e., $f>0$.

We extend the interpolation so that it can capture two discontinuities: a strong discontinuity across the crack surface which can be represented with a jump in displacements and a weak discontinuity which can be considered as a jump in strains. It can be shown that when the interpolation reproduces independent linear functions on sides of the interface, then such discontinuities can be captured. Therefore, the interpolation is enriched to reproduce two discontinuous displacement fields: a displacement field with a strong discontinuity denoted by Φ^{u} which is defined using the Heaviside function, i.e.,

$$\Phi^{\mathrm{u}} = H(f(\mathbf{X})) = \begin{cases} 0 & \text{if } f<0, \\ 1 & \text{if } f>0, \end{cases} \tag{2}$$

and another displacement field with a weak discontinuity, i.e., a discontinuity in its first derivative denoted by $\Phi^{\nabla\mathrm{u}}$ which is defined as

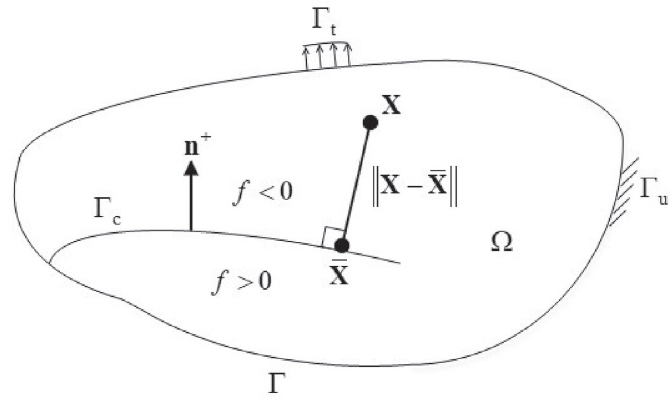

Fig. 1. A two-dimensional body with a crack in material coordinates.

$$\Phi^{\nabla\mathrm{u}} = H(f(\mathbf{X}))\times f(\mathbf{X}) = \begin{cases} 0 & \text{if } f<0, \\ f(\mathbf{X}) & \text{if } f>0. \end{cases} \tag{3}$$

For each discontinuity, a physically – based variable which can best reflect the nature of the discontinuity is defined as an enrichment parameter and assigned to non-nodal points on the surface of discontinuity. In the following, we first construct the enrichments for a linear element in one dimension and then for a linear triangular element in two dimensions.

2.1. Representation of a crack with non-nodal enrichment parameters for 2-node linear elements

We consider a one-dimensional bar with a discontinuity, i.e., a crack at $X=X^c$ as shown in Fig. 2. The level set function f is considered negative and positive on the left and right sides of the crack, respectively. Fig. 2(a) illustrates an arbitrary displacement field consisting of two independent linear fields on each side of the crack. Such displacements can be reproduced by superimposing three independent parts: (i) a continuous displacement represented by the finite element interpolation shown in Fig. 2(b); (ii) a strong discontinuity represented by a jump in the displacement $[\![u]\!]$ at the non-nodal point at X^c as illustrated in Fig. 2(c); and (iii) a weak discontinuity represented by a jump in the strain $[\![L^e\nabla u]\!]$ at X^c as described in Fig. 2(d). Here, the element length L^e is consciously multiplied to make the units of nodal values consistent with other terms in (4). This results in significant reduction of the condition number of the stiffness matrix. Notice that four unknown variables are employed to approximate a displacement field in this bar, i.e., $\{u_1, u_2, [\![u]\!], [\![L^e\nabla u]\!]\}$.

Using these variables, the approximation of the displacement field in a one-dimensional bar can be defined by

$$\begin{aligned} u(X,t) &= u^{\mathrm{cont}}(X,t) + u^{\mathrm{disc}}(X,t) \\ &= \sum_{I=1}^{2} N_I(X)\,u_I(t) + \Psi^{\mathrm{u}}(X)\,[\![u(t)]\!] + \Psi^{\nabla\mathrm{u}}(X)\,[\![L^e\nabla u(t)]\!]. \end{aligned} \tag{4}$$

The non-nodal enrichment functions Ψ^{u} and $\Psi^{\nabla\mathrm{u}}$ are constructed so that (4) can reproduce the discontinuous displacement fields Φ^{u} and $\Phi^{\nabla\mathrm{u}}$ given in (2) and (3), respectively. These discontinuous fields are illustrated in Fig. 3.

Let us first consider the displacement field with a strong discontinuity, i.e., $u=\Phi^{\mathrm{u}}$. Then, the nodal values consisting of the regular and enriched degrees of freedom (DOFs) can be calculated as

$$\text{Regular DOFs}: \begin{cases} u_1 = 0 \\ u_2 = 1 \end{cases}, \qquad \text{Enriched DOFs}: \begin{cases} [\![u]\!] = 1 \\ [\![L^e\nabla u]\!] = 0. \end{cases}$$