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G^1 Bézier surface interpolation with T-junctions at a 3-valent singular vertex ☆

Min-Jae Oh [a], Myung-Il Roh [a,b], Tae-wan Kim [a,b,*]

[a] Research Institute of Marine Systems Engineering, Seoul National University, Seoul 08826, Republic of Korea
[b] Department of Naval Architecture and Ocean Engineering, Seoul National University, Seoul 08826, Republic of Korea

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A B S T R A C T

A singular vertex cannot always be avoided in constructing networks of boundary curves. When three boundary curves, two tangents of which are collinear, meet at a singular vertex, then the region that is bounded by the curves makes T-shape. We give a necessary condition for G^1 interpolation of a 3-valent singular vertex with Bézier surfaces, suggest a subdivision method with three rectangular sub-patches including T-junctions for use when this condition is not met, and give examples of its use.

1. Introduction

Surface interpolation from a given boundary curve network is a common method for free-form surface modeling in current CAD and graphics systems. Smooth surfaces can easily be generated from well-designed boundary curves. Thus designers try to create networks of boundary curves that do not have unusual characteristics such as misaligned patch edges (Polini and Kuutti, 2009). However, these are sometimes unavoidable. In this paper, we present a surface interpolation method for a boundary curve network composed of Bézier curves with a singular vertex. A T-junction can occur in a network of boundary curves when a boundary curve of a rectangular region ends in the middle of the boundary of another region (Fig. 1a), or a 3-valent singular vertex can occur when the boundary curves of three regions meet at a vertex, and two of them have the same tangent direction (Fig. 1b). In this case the patch covering one of the regions will be singular and the surface normal at the singular vertex is undefined. However, it can be found from adjacent patches for smooth interpolation. We will refer to the topology shown in Fig. 1a as a *T-junction at a patch edge*, and to that in Fig. 1b as a *3-valent singular vertex*. The shape of the singular patch is a triangle, therefore, a triangular representation such as Bézier triangle can be used for the patch, but it approximates the boundary curves. However, in this study, we will obtain exact interpolation using three rectangular subdivision rather than an approximation. Also, many CAD systems don't allow the triangular patch, and the triangular patch is converted into rectangular patch by merging two corners by a designer. However, in case of surface analysis or mesh generation, it is not recommended because the rectangular patch with triangular shape can cause

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* Corresponding author at: Department of Naval Architecture and Ocean Engineering, Seoul National University, Seoul 08826, Republic of Korea.
E-mail address: taewan@snu.ac.kr (T.-w. Kim).
URL: http://caditlab.snu.ac.kr (T.-w. Kim).

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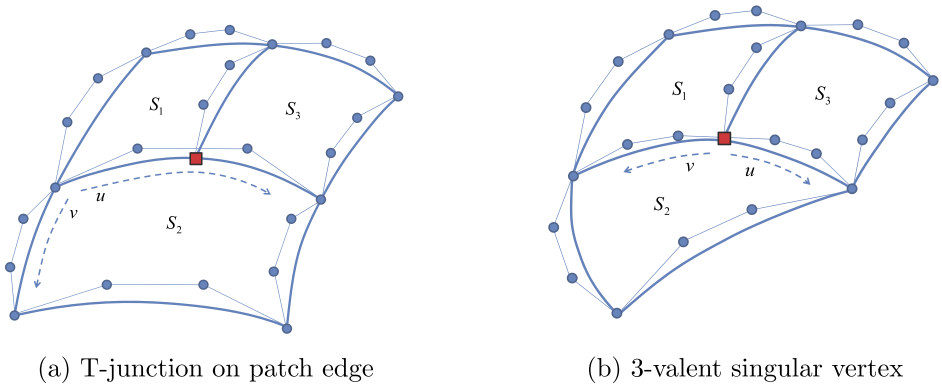

Fig. 1. T-junction and singular vertex (square symbol) (Oh et al., 2012).

bad resolution of the surface. Therefore, the Bézier triangle is not a solution for the problem. A 3-valent singular vertex is not common in CAD modeling, but the mathematical property for interpolating the singular vertex with G^1 continuity should be discussed. A surface interpolation method for a T-junction on a patch edge has been presented in a previous paper (Oh et al., 2012). In this paper, we establish the necessary condition to construct a G^1 surface with a singular patch, and propose a subdivision method to construct G^1 surfaces with a 3-valent singular vertex. We are not using the word "subdivision" to refer to recursive subdivision (Catmull and Clark, 1978), but to a procedure similar to that employed by Shirman and Séquin (1987, 1991) to overcome the twist incompatibility that occurs on singular boundary curves.

1.1. Previous work

A lot of work has been done on smooth surface interpolation from a given boundary curve network (Farin, 1982; Farin et al., 2002; Hanmann and Bonneau, 2000). Degen (1990) determined explicit conditions for C^1 and C^2 continuity between adjacent Bézier patches. Hermann and Lukács (1996) extended Degen's method to the G^n continuity of polynomial surfaces. Loop (1994) devised a way to construct a C^2 boundary curve network of arbitrary topological type and introduced the twist compatibility condition. We will use this condition to establish the necessary condition to construct G^1 surfaces at a singular vertex. Farin et al. (2002) analyzed the geometric continuity of curves and surfaces and established vertex enclosure constraints for G^k continuity. Several other authors (Sarraga, 1987; Peters, 1991; Liu and Sun, 1994; Cho et al., 2007) have demonstrated that given boundary curves need to satisfy collinearity and co-planarity constraints to achieve G^1 continuity at an ordinary-valent vertex where four patches meet: the four tangents to the incident curves at a 4-valent vertex should be pairwise-collinear. Hermann et al. (2011) introduced a vertex enclosure constraint for curve networks with C^1 continuity and analyzed 4-valent vertex which does not form 'X' (pairwise-collinear). Farin (2002) and Sarraga (1987) developed the condition for G^1 continuity in terms of the Bézier control points of adjacent patches. Building on these methods, Cho et al. (2006) have proposed a method of network interpolation for ship hull design in which is filled G^1 Bézier patches; a network, which contain 3, 4, and 5-sided regions; they analyzed the degree of the patches required, and the local singularities that can occur during construction (Cho et al., 2007). However, several different subdivision strategies are required to preprocess curves with T-junctions. Tong and Kim (2009a, 2009b) introduced a local method of generating a singularity-free G^1 triangular spline surface with a minimum degree of 6. A triangular network with an n-valent vertex topology is created T-junctions whom are removed by triangular subdivision. Oh et al. (2012) presented a method of G^1 surface interpolation for a T-junction on a patch. They create an auxiliary off-boundary curve where the two surfaces meet at the T-junction. We develop this idea to become a rectangular subdivision method which introduces two additional T-junctions. In this paper, the biquintic Bézier surfaces are used to interpolate the given boundary curves same as in the research (Oh et al., 2012). Lower degree surfaces such as biquadratic or bicubic were proposed by Gregory and Zhou (1994) and Peters (1992). However, John et al. suggested a perturbing some of the boundary data and further partitioning methods to avoid the singularity problem from the lack of free variables. Also Perters used the bivariate mesh of points for the surface construction. This means that the boundary curve can be changed to satisfy the continuity constraints when the low degree surfaces are used for interpolation. In this paper, it is assumed that the boundary curves should not be changed. The singularity analysis due to the degrees of given boundary curves, surfaces, and weight functions are described in Cho et al. (2007).

1.2. Main result

A 3-valent singular vertex occurs when three patches meet at a single patch corner and two of the three boundary curves meet tangentially. We establish the necessary condition for generating G^1 surfaces at such a singular vertex by formulating a generalized (algebraic) G^1 continuity constraint. This condition tells us about the existence of G^1 surfaces interpolating