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# Theoretical prediction for maximum residual cross-sectional deformation of thin-walled cylindrical steel tubes under pure plastic bending

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ABSTRACT

Cross-sectional ovalization ( ovalisation ) usually occurs when thin-walled tube is subjected to large plastic bending. This paper is concerned with residual deformation of tube's cross-section in radial direction when external bending moment is removed. A combination of experiments and theoretical modeling is used to study this issue. Firstly the four-point pure bending experiments involving fifteen stainless steel specimens with a range of diameter-to-thickness ratios are carried out to observe the shape of the ovalized cross-section and to obtain the cross-sectional flattening (value of ovalization) along the circumferential direction. Secondly the theoretical modeling procedure is performed to derive a rational model for predicting the maximal residual cross-sectional flattening of the thin-walled tube subjected to pure bending after unloading process, employing the thin-shell kinematics, the principle of virtual work and the classic unloading rule. Finally the model is validated by comparing the theoretical results with the experimental ones with less than 10% error. And the relationships between the residual flattening and the bending radius as well as the wall thickness are also revealed by the numerical results. The application of this investigation will play a positive role in thin-walled steel tube bending operation.

## 1. Introduction

The high-precision circular cylindrical shells, referred to as thin-walled tubes or pipes, have a number of applications in offshore or aerospace structures, nuclear reactor components, petrochemical and military industry. During service, the tubes usually undergo large plastic bending, which causes cross-sectional deformation. This deformation is characterized by a localized cross-sectional ovalization due to the interaction of the compressive and tensile longitudinal stresses on opposite sides of the neutral surface during bending process. And the cross-section will become more oval as the bending curvature increases. When the bending loads have been removed, there is residual cross-section deformation after springback, which will reduce the tube rigidity and limit the extent to which the tube can be deformed or loaded. That is known to have great effects on the stability of the structure. In order to keep enough rigidity and control the roundness, the maximum residual cross-sectional flattening (value of ovalization, is equal to the length change of the diameter before and after bending) after unloading process should be controlled under an allowed limit. So the first requirement from the analysis of this issue is the capability of predicting the maximum residual flattening accurately.

Since Brazier [1] employed a simplified method to describe the nonlinear cross-sectional ovalization phenomenon observed in bending experiments at first. A lot of work relating to the ovalization behavior of thin-walled cylindrical tubes under pure bendings has been done. Some of them are cited here. Li [2] extended the Brazier's method to the nonlinear collapse of orthotropic composite cylinder under pure bending, and presented a set of closed-form formulations to predict both critical dynamic and static loads. Houliara and Karamanos [3] studied the structural stability of long uniformly elastic cylinder shells under in-plane bending to detect the bifurcation on the pre-buckling path and trace the post-buckling path by non-linear FEA method. The relationships between internal or external pressure, radius-to-thickness ratio, as well as initial curvature and ovality and bifurcation wavelength, corresponding curvature as well as critical moment were obtained. Ziso and Shoham [4] developed a model for general cross-section shapes of thin-wall tubes, which was solved by elastic energy theory based on Karamanos's and Brazier's instability model, to predict the critical curvature, moment as well as flattening when the buckling in thin-walled tube under pure bending had occurred. However, All the preceding studies focused on the material that undergoes elastic deformation. And there has been much more work done in plastic range in order to describe the nonlinear response of cylindrical tubes under large plastic bending.

Ades [5] firstly presented a numerical method to analyze the tube's ovalization using the energy principle in the plastic range. Then Gellin

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[6] proposed a more accurate formulation to this issue applying $J_2$ deformation theory of plasticity and improved kinematics. Then his formulation was improved by Shaw and Kyriakides[7] using flow theory of plasticity. Zhang and Yu [8] proposed a general model dealing with the ovalization for the tube of arbitrary cross-sectional shape. Tatting et al. [10,9] employed the semi-membrane constitutive theory to establish a set of governing equations, which could exhibit the nonlinear bending response for finite length composite tubes considering the cross-sectional ovalization. Hiroyuki et al. [11] used thin-shell theory to propose a model, which could describe the cross-sectional ovalization for multilayered circular cylinders under pure bending. The above studies for the tube's ovalization in the plastic range compared well with experiments. However, most of them are restricted to analyze relatively small degree of ovalization owing to neglecting higher-order terms in the kinematics.

In recent years more and more researchers concerned with the buckling response of circular cylindrical shells subjected to bending loads. Munz and Mattheck [12] considered that the main reason of cross-section ovalization was the vertical component of the axial bending stresses. Following this Chitkara and Prinja and Chitkara [13] proposed a theoretical solution of the post-collapse cross-sectional flattening for thick pipes under plastic bending, using four plastic hinges to model the circular cross-section of the tube. Kyriakides and Ju [15], Ju and Kyriakides [14] tested aluminum 6061-T6 shells with 11 different diameter-to-thickness ratios ranging from 19.5 to 60.5. According to the experimental results, they divided the shells into three broad categories, and proved that the major deformation characteristic for such shells was the cross-sectional ovalization induced by pure bending. And then they proposed a series of formulations to predict the response and various instabilities observed in the experiments using Sanders' shell kinematics and the principle of virtual work, the numerical results agreed well with the experimental results. Wierzbicki and Sinmao Monique [16] employed four travelling hinges to model cross-sectional ovalization for the tube under large plastic bending, and predicted cross-sectional deformation and bending moment using a simplified theoretical model. Li and Kettle [17] investigated the nonlinear bending response of finite length cylindrical shells with stiffening rings, which was derived by the shell kinematic relations and the minimum potential energy principle. Then they determined the corresponding critical moment associated with local buckling, employing the Seide-Weingarten approximation. Houliara and Karamanos [3] developed a simplified closed-form analytical expressions for the bifurcation curvature and the buckling moment, as well as for the corresponding buckling wavelength, based on the non-linear DMV shell equations, when they investigated the structural stability of long uniformly thin elastic tubular shells subjected to in-plane bending. Using this simple mechanical model, they illustrated the pre-buckling ovalized state and the effects of pressure, the bifurcation response of tubes in a qualitative manner. Elchalakani et al. [18] derived a closed-form solution to exhibit moment-rotation response of the tube subjected to pure bending using the Mamalis kinematics and two local plastic mechanisms (diamond and star shapes). Then Poonaya et al. [19] improved Elchalakani's model considering the rate of energy dissipation on the rolling hinge in the circumferential direction, based on the principle of energy rate conservation. Mentella and Strano [20] predicted the magnitude of cross-sectional deformation for hydraulic tubes during the rotary draw bending process, also using four travelling hinges to model cross-sectional ovalization. Referring to Tomasz Wierzbicki's model of the cross-sectional flattening, Ji et al. [21] established the energy rates of the ovalized tube and the ovalizing tube during bending process to analyze the buckling response using the rigid-perfectly plastic material model, then they proposed an expression to obtain apparent strain of a tube under bending when the plastic buckling had occurred. Ciprian [22] discussed the bifurcation instabilities in finite pure bending for circular cylindrical shells when the progressive flattening of the cylindrical cross-section was explicitly taken into account, he defined the main governing equations for the possible bifurcations of the pure bending problem, using the well-known Donnell-Mushtari-Vlasov (DMV) shallow-shell buckling equation. And the singular perturbation methods were used to obtain the simple asymptotic approximations for the critical curvature and bending moment associated with the bifurcations. However, the models mentioned above are usually used to analyze the buckling response, which occurs in a limit case under bending. The tube cross-sectional shape at the buckling state is different in that at the general plastic bending state. So these models are valid only for large sectional distortion, and not suitable for the plastic bending without buckling.

All the above researches focus on the loading process of the tube subjected to the bending loads, but the residual deformation appears in unloading process. And the precise prediction and accurate control of springback are always the key challenges in bending of tubes. Many researchers have worked on them, only a little of their work relating to this paper is discussed here. The cross-sectional springback and the flattening behavior for the tube subjected to pure bending were firstly investigated by Mori et al. [23] using 4-point bending experiments, which were carried out on adhesive-bonded two-layer clad tubes consisting of copper, hard aluminum and soft aluminum, to obtain the continuous measurements of the bending curvature and the residual flattening along the radial direction during deformation. Thuvander [24] investigated the bearing ring's out of roundness under bending experimentally and numerically, measured and calculated the distortion of outside profile of the tube cross-section after three point bending. E and Liu [25]explored the springback of 1Cr18Ni9Ti stainless steel tubes in rotary draw bending tests, and attempted to derive the formulations of total springback, time-independent springback and time-dependent springback. Then Liu and E [26] presented an analytic model to predict the tangential strain and radial springback of the tube under bending considering cross-sectional ovalization. This model could better predict the relationship between wall thickness ratio, bending radius ratio and springback angle, comparing with the calculation neglecting ovalization. Li et al.[27] studied the deformation theory of plasticity, explicit/implicit FE method and experimental approaches on medium strength 6061-T4 Al-alloy thin-walled tubes in universal rotary draw bending, explored and clarified the nonlinear springback rules and mechanisms regarding angular springback and radius growth by changing the tube geometric parameters. Mehdi and Ali [28] employed the FE simulations to predict the springback of titanium alloy (Ti-3Al-2.5 V) tubes under bending using various material hardening models, and obtained the influence of the mandrel on the tube cross-section deformation and springback.

In the above studies for the prediction and control of the springback of tubes under bending, most analytic models and experimental methods are all presented to predict the **axial springback** after bending only considering cross-sectional ovalization, but not focus on **the circumferential** springback. so they can not be used to calculate the residual flattening of the tube cross-section under bending.

The present study focuses on the ovalized state of thin-walled tube cross-section in loading and unloading process under pure bending without bifurcation buckling which is characterized by periodic waves on the tube compressed side. The main objectives of the study are to obtain a set of theoretical formulations to predict the maximum residual flattening of thin-walled tube cross-section after unloading process, to reveal the major deformation characteristic of the tube cross-section due to ***Brazier effect*** after pure bending, and to analyze the major factors which influence the residual cross-sectional flattening. So the 4-point bending experiments will be carried out on stainless steel tubes with different diameter-to-thickness ratios under different bending radii. And a set of theoretical formulations will be proposed to model the deformation process and predict the residual flattening. The validity of the formulation will be verified by direct comparison with the experimental results. And the influences of the bending radius and the wall thickness on the residual flattening are also analyzed.