Thin-Walled Structures 133 (2018) 226–234

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Unified parametric modeling of origami-based tube

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ARTICLE INFO

Keywords:
Unified
Parameterization
Modeling
Origami
Tube

ABSTRACT

Existing parameterization of origami structures is based on angles, which is an inconvenience for the fabrication and calculation of the geometrical properties of an origami-engineering project. In this paper, a unified parameterization based on the dimensional parameters, including the length and height instead of the angles, is proposed. Various origami-based tubes reconstructed through the proposed parameterization process and compiled into a program are then described. Three types of tubes including non-flat foldable, locally flat-foldable, and entirely flat-foldable are systematically divided when forming a closed-loop tube, and a criterion is proposed for estimating the flat foldability of the tube. Finally, the proposed theory is validated based on physical prototypes folded using paperboard and fabricated using polymer through a blow molding process. The constructed origami-based tubes provide extensive guidance for the design of energy absorbing structures.

1. Introduction

The crashworthiness of a device is of considerable importance to ensure the safety of passengers for a transport vehicle design [1–3]. A tube with presupposed corrugations on its surface is used as an energy absorbing device [4,5]. When compressed or impacted, a tube is able to deform along the corrugations, thereby reducing the peak load and expending more energy by enlarging the mean load [4,6]. A tube can be used for various applications, such as a crash box [7] or vehicle bumper [8], as well as in the landing gear of a helicopter [9].

Origami is a topic of active research in the scientific community, and is used to create various folded devices. Origami-based tubes have a unique characteristic in terms of the crease arrangement, leading to a foldable deformation during compression [10–12]. On the one hand, they can be used as a nearly kinematic mechanism, focusing on the kinematic analysis of the mechanism applied [12,13]. On the other hand, they can be used as a mechanical structure, focusing on the collapse behavior of the energy absorption [7,14].

The geometric modeling approaches used in these studies vary widely because the particular geometric parameters are typically only derived as needed [15]. Most studies on origami-based tubes have been parameterized in term of their angles, for example, an edge angle [13] and a folding angle [12,16]. When identifying and exploring their geometrical properties, a complex conversion of the angles into the dimensions is required, for example, the length [10,17], volume [18], and density [19]. However, in engineering studies and actual fabrication, patterns have often been denoted in terms of their dimensions, which allow a straightforward calculation of the geometrical properties, for example, the height [14,20,21], and are directly available for fabrication without the above conversion, for example, the total length [22,23] and height [7,24]. Consequently, when exploring patterns for the engineering and fabrication of an origami structure, existing parameterizations in terms of the angles appear to be inconvenient and incomplete. It is important to highlight that a study concerning the parameterization in terms of dimensions can be applied to identify the mechanical properties of an origami structure.

This paper focuses on unified parameterized modeling depending on the dimensional parameters, including the length and height instead of the angles. First, a general crease pattern consisting of various isosceles trapezoids as the basic geometrical units is proposed. Next, a transformation from a crease pattern to its corresponding origami-based tube is parameterized. Various types of tubes compiled through a systematic analysis of their length and height are then described. In addition, a classification of the tubes is presented in terms of their flat foldable feature, and a criterion is discovered to estimate their flat foldability. Finally, physical tubes folded from paperboard and fabricated using polymeric blowing form are utilized to verify the models established through the proposed parameterization. The constructed tubes provide extensive guidance for the design of energy absorbing structures.

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https://doi.org/10.1016/j.tws.2018.09.043
Received 7 June 2018; Received in revised form 20 September 2018; Accepted 26 September 2018

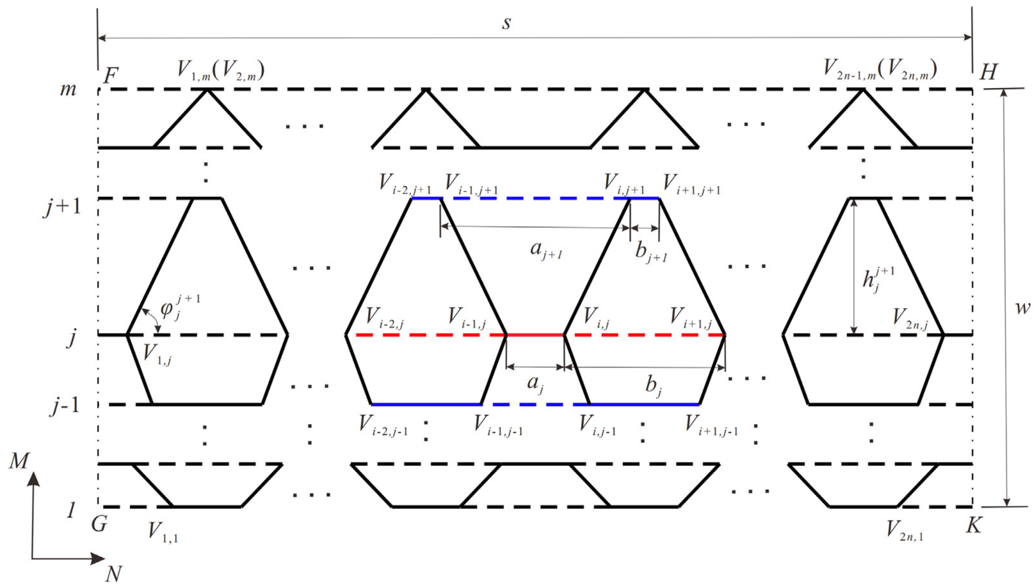

Fig. 1. Crease pattern at the deployed state and parametric definition. Mountain and valley fold lines in the crease pattern are represented using heavy solid and dashed lines, respectively.

2. Modeling of origami-based tube

It is well known that Miura and Yoshimura patterns are composed of identical parallelograms and triangles as the basic geometric units, respectively. Herein, a general origami crease pattern is proposed, as shown in Fig. 1, in which the creases consist of edges of isosceles trapezoids as the basic geometrical units. Many layers are contained in the crease pattern, and each trapezoid is connected with its adjacent trapezoid through shared edges. Various shapes of the trapezoids can be exhibited through an adjustment of the deployed heights, as well as the upper and lower bases. According to the definition of rigid foldable origami, the facets applied in this study are flat and cannot be stretched or ripped during folding, in which fold bends should be conducted along the creases [25]. In addition, we focus on the parameterized transformation, from the initial crease pattern to its corresponding origami-based tube, instead of a kinematic analysis of the foldability and energy absorbing capability of the structures.

2.1. Definition of crease pattern

As shown in Fig. 1, the total length and width of one crease pattern are usually given, and are denoted as s and w, respectively. The crease pattern starts from the midpoint of the base of one trapezoid, and has $2n$ creases and m layers along the $N-$ and $M-$ directions, respectively. Herein, vertexes are numbered as $1, 2, \ldots, i, \ldots, 2n$ ($n \geq 2$) along the $N-$ direction, and layers are numbered as $1, 2, \ldots, j, \ldots, m$ ($m \geq 2$) along the $M-$ direction. The vertex $V_{i,j}$ thus represents the i^{th} point in the j^{th} layer ($i = 1, 2 \ldots 2n$, $j = 1, 2 \ldots m$). The j^{th} layer consists of an alternating assignment of n edges of a_j, and n edges of b_j, without a loss of generality. Two bases of the isosceles trapezoid can be denoted, utilizing the edge lengths of a_{j+1} and a_j, or b_{j+1} and b_j. Furthermore, the edge lengths a_j are independent in different layers ($j = 1 \ldots m$). The deployed heights of trapezoids in the crease pattern, denoted as h_j^{j+1} ($j = 1, 2 \ldots m-1$), are equivalent in the same layer, and are independent between different layers. Inner angles of trapezoids are denoted as φ_j^{j+1} ($j = 1, 2 \ldots m-1$), which are assumed to fall within the range of $\left(0, \frac{\pi}{2}\right)$. The inner angle related to the shapes of the trapezoid is not exactly preserved. As a result, the total length (s), total width (w), and inner angle (φ_j^{j+1}) in a crease pattern can be expressed as follows:

$$s = n(a_j + b_j) = n(a_{j+1} + b_{j+1}) = n \cdot l \tag{1}$$

$$w = \sum_1^{m-1} h_j^{j+1} \tag{2}$$

$$\varphi_j^{j+1} = \arctan\left(\frac{2h_j^{j+1}}{|a_j - a_{j+1}|}\right) \tag{3}$$

where l indicates the length of one periodic segment of the total length.

2.2. Verification of coplanarity

During folding, the two-dimensional (2D) crease pattern is transformed into a three-dimensional (3D) configuration. Each vertex $V_{i,j}$ is surrounded by four trapezoids in the crease pattern and has singly folding track. Herein, four adjacent vertexes ($V_{i-2,j}$, $V_{i-1,j}$, $V_{i,j}$, and $V_{i+1,j}$) and their corresponding trapezoids are selected to analyze their folding locations without of a loss of generality, as shown in Fig. 2a. At an arbitrary folding state, eight dihedral angles between plane $V_{i-1,j+1}V_{i,j+1}V_{i,j}V_{i-1,j}$ and plane $V_{i-2,j}V_{i-1,j}V_{i,j}$, between plane $V_{i-1,j+1}V_{i,j+1}V_{i,j}V_{i-1,j}$ and plane $V_{i-1,j}V_{i,j}V_{i+1,j}$, between plane $V_{i-1,j-1}V_{i,j-1}V_{i,j}V_{i-1,j}$ and plane $V_{i-1,j}V_{i,j}V_{i+1,j}$, between plane $V_{i,j+1}V_{i,j}V_{i+1,j}$ and plane $V_{i-1,j}V_{i,j}V_{i+1,j}$, between plane $V_{i-1,j+1}V_{i,j+1}V_{i,j}V_{i-1,j}$ and plane $V_{i-1,j}V_{i,j}V_{i,j-1}V_{i-1,j-1}$, between plane $V_{i-1,j}V_{i,j}V_{i,j-1}V_{i-1,j-1}$ and plane $V_{i,j-1}V_{i,j}V_{i+1,j}$, between plane $V_{i,j+1}V_{i,j}V_{i+1,j}$ and plane $V_{i+1,j}V_{i,j}V_{i,j-1}$, and between plane $V_{i,j+1}V_{i,j}V_{i+1,j}$ and plane $V_{i-1,j+1}V_{i,j+1}V_{i,j}V_{i-1,j}$, are denoted as $\lambda^1_{i-1,j}$, $\lambda^1_{i,j}$, $\lambda^2_{i,j}$, $\lambda^6_{i,j}$, $\delta^1_{i,j}$, $\delta^2_{i,j}$, $\delta^3_{i,j}$, and $\delta^4_{i,j}$, respectively. The former four dihedral angles are assumed to fall within the range of $[0, \frac{\pi}{2}]$. The last four angles are assumed to fall within the range of $[0, \pi]$. Two edge angles $\angle V_{i-2,j}V_{i-1,j}V_{i,j}$ and are denoted as $\varepsilon^1_{i-1,j}$ and $\varepsilon^1_{i,j}$, respectively, which are assumed to fall within the range of $[0, \pi]$. After one unit sphere around vertex $V_{i,j}$ is used to cut four trapezoids around the vertex $V_{i,j}$ in Fig. 2a, two spherical triangles make up of four sectors of single-vertex which is cut from the trapezoids, as shown in Fig. 2b. The angles and edges of the spherical triangles can be marked. Applying the sine and cosine laws of spherical triangles, three relationships can be established between dihedral and edge angles as follows (see the Appendix A for details):