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Arbitrarily shaped plates analysis via Line Element-Less Method (LEM)

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ABSTRACT

An innovative procedure is introduced for the analysis of arbitrarily shaped thin plates with various boundary conditions and under generic transverse loading conditions. Framed into Line Element-less Method, a truly meshfree method, this novel approach yields the solution in terms of the deflection function in a straightforward manner, without resorting to any discretization, neither in the domain nor on the boundary. Specifically, expressing the deflection function through a series expansion in terms of harmonic polynomials, it is shown that the proposed method requires only the evaluation of line integrals along the boundary parametric equation. Further, minimization of appropriately introduced novel functionals directly leads to simple systems of linear algebraic equations for the unknown expansion coefficients. Notably, the proposed procedure yields exact solutions, when available, for different plate geometries. Additionally, several numerical applications are presented to show the reliability and simplicity of the approach, and comparisons with pertinent Finite Element method data demonstrate the efficiency and accuracy of the proposed procedure.

1. Introduction

Many structural problems in engineering mechanics are governed by partial differential equations (PDEs) whose exact solutions is known for few restricted cases of practical interest. In this regard, the evaluation of the structural response of plates under generic loading conditions, commonly described via a biharmonic PDE in Kirchoff's theory, is a well-established problem in applied mechanics due to the constant use of these structural elements in most engineering fields.

Clearly, since exact plate solutions are available only for certain shapes, boundary and loading conditions [1], several numerical procedures have been proposed and their development still attracts the attentions of many researchers in the field [2–4]. In this context, the Finite Element Method (FEM) [5] and Boundary Element Method (BEM) [6] unquestionably represent the most commonly employed and powerful numerical techniques for general structural analysis.

As well-known the use of mesh, be it in the domain or in the boundary, is a common characteristic of these traditional approaches. Specifically, while conventional FEM approach basically requires a discretization over the entire domain through finite elements mesh, in the BEM an integral equation is obtained and a boundary mesh is required to numerically approximate the boundary integrals involved. It is worth underscoring that, in this latter approach the governing differential equation is satisfied exactly inside the domain and high accuracy is generally achieved with a relatively small number of boundary elements. Notably, the extensive research efforts devoted in the last few decades to the development of these approaches have allowed to circumvent most numerical problems associated to the domain or boundary discretization, thus making FEM and BEM the dominant approaches for most problems in computational mechanics.

Nevertheless, the possibility of obtaining numerical solutions for PDEs without resorting to any discretization, that is the so-called meshless approach, has rather recently gained the attention of scientists and engineers working in this field. As defined in [7] a meshless method, also referred to as meshfree method, is a method used to establish system equations for the whole problem domain without the use of a predefined mesh for the domain discretization. This approach has, therefore, become an alternative to classical FEM and BEM due to some beneficial features such as its flexibility, wide applicability and the possibility of avoiding problems related to meshing and remeshing in the domain or boundary [7,8].

In this regard, framed in the meshless approach, different procedures have been proposed to solve a variety of engineering problems [9], such as the element free Galerkin Method [10,11], Petrov–Galerkin

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approach [12], h-p clouds method [13], and the reproducing kernel element method [14] among the others. Further, specifically referring to the plate analysis, the works in [15–22] and references therein can be mentioned. Finally, note that other classes of methods, which are inherently meshless, exist for the plate bending problem, including the Trefftz method [23], the pb-2 Rayleigh-Ritz method [24,25], and the Galerkin method [26].

Additionally, a novel truly meshless procedure, namely the Line Element-less Method (LEM), has been introduced for the analysis of De Saint Venant pure torsion and flexure-torsion problem for both isotropic and orthotropic material [27–31]. Notably, this method does not require any discretization neither in the domain nor on the boundary, and all the involved integrals are simple line integrals. Further, based on the analogy between plates bending under edge moments and beams in torsion [32–34], recently the aforementioned LEM has been employed for the bending problem of simply supported plates subject to uniformly distributed edge moments [35].

In this context, aim of this paper is to extend the LEM for the analysis of arbitrarily shaped plates, without any holes, assuming various boundary conditions (BCs) and subject to transverse loads. Specifically, the original biharmonic PDE, which rules the plate deflection, is decomposed in two Poisson's equations, whose solution is expressed as the superposition of pertinent particular solution and harmonic polynomials with unknown expansion coefficients. These coefficients are then determined satisfying the prescribed BCs on the contour.

Note that, in the proposed procedure the BCs are satisfied in a least square sense on the plate contour, and only line integrals along the boundary parametric equation are required, leading to systems of linear algebraic equations for the unknown expansion coefficients.

Remarkably, as it will be shown in the following, this procedure yields exact closed-form solutions when available, for different plates geometries, while in the other cases approximate accurate analytical solutions are achieved generally employing few terms in the series expansion. This may be clearly considered an attractive feature of the proposed method, especially with respect to other meshfree procedure which are inherently exclusively numerical in nature.

Interestingly, unlike the Trefftz method [23], where the BCs are enforced in a number of boundary points to determine pertinent expansion coefficients, or classical meshfree approaches, where several nodes are generally considered in the domain, this proposed procedure is entirely element-free. Further, with respect to the classical Rayleigh-Ritz approach [24,25] more general plate shapes and BCs can be handled, and cumbersome integration over domains are not involved. These aspects may clearly represent an advantage of the proposed procedure.

Several numerical applications will be shown, demonstrating the elegance and simplicity of the proposed procedure, and corresponding data vis-à-vis classical FEM results will be reported, assessing the accuracy and reliability of the procedure.

2. Problem definition

Consider a homogeneous isotropic thin plate, of arbitrary shape with contour Γ and domain Ω, uniform thickness h and modulus of elasticity E, generally referred to as Kirchhoff plate (see Fig. 1). The governing differential equation in terms of transverse deflection $w(x, y)$ is the well-known biharmonic Eqs. [1,36]

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=\frac{q(x, y)}{D} \tag{1}$$

where $q(x, y)$ is the transverse distributed load, $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate and ν is the Poisson ratio.

The bending moments $M_x(x, y)$ and $M_y(x, y)$, and the twisting moment $M_{xy}(x, y)$ are given as

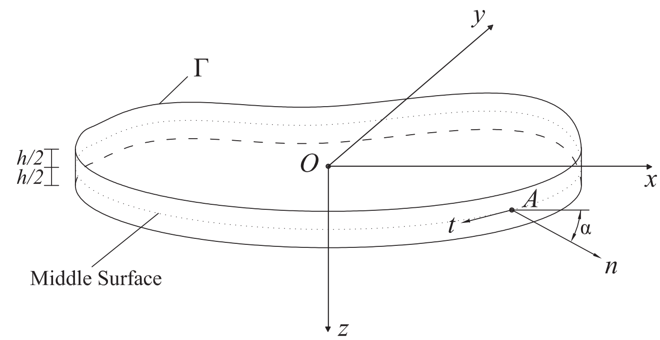

Fig. 1. Plate with arbitrary shape.

$$M_x(x, y) = -D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \tag{2.a}$$

$$M_y(x, y) = -D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right) \tag{2.b}$$

$$M_{xy}(x, y) = -D(1-\nu)\frac{\partial^2 w}{\partial x\,\partial y} \tag{2.c}$$

while the shearing forces $V_x(x, y)$ and $V_y(x, y)$ are given by

$$V_x(x, y) = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{3.a}$$

$$V_y(x, y) = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{3.b}$$

Further, introducing the so-called moment sum $M(x, y)$ as

$$M(x, y) = \frac{M_x+M_y}{1+\nu} \tag{4}$$

Eq. (1) can be recast into two equivalent Poisson's equations as [37]

$$\nabla^2 M(x, y) = -q(x, y) \tag{5.a}$$

and

$$\nabla^2 w(x, y) = -\frac{M(x, y)}{D} \tag{5.b}$$

where $\nabla^2(\cdot) = \frac{\partial^2(\cdot)}{\partial x^2}+\frac{\partial^2(\cdot)}{\partial y^2}$ is the well-known Laplace operator.

Thus, the solution of the plate problem Eq. (1) reduces to the integration of the two Eqs. (5 a, b) in succession, which is sometimes preferred depending upon the method of solution employed.

As far as the boundary conditions (BCs) are concerned, denote as n and t the outward unit normal and tangent vector at a point A of a generic curvilinear edge of the contour Γ, and let α be the angle between the normal n and the x axis (see Fig. 1). Thus, for the most common cases, the boundary conditions for the curvilinear edge can be specified as [38–40]

i. Simply-supported edge

$$w(x, y) = 0 \tag{6.a}$$

$$M_n(x, y) = 0 \tag{6.b}$$

where $M_n(x, y)$ denotes the normal bending moment applied at the edge, and is given as

$$M_n(x, y) = n_x^2\,M_x + n_y^2\,M_y + 2\,n_x\,n_y\,M_{xy} \tag{7}$$

where n_x and n_y are the components of the unitary vector n along the x and y axes, respectively.