Journal of Constructional Steel Research 151 (2018) 216–227

Contents lists available at ScienceDirect

Journal of Constructional Steel Research

Formulation for second-order inelastic analysis of steel frames including shear deformation effect

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ARTICLE INFO

Article history:
Received 15 April 2018
Received in revised form 28 August 2018
Accepted 12 September 2018
Available online xxxx

Keywords:
Shear deformation
Timoshenko theory
Spread of plasticity
Inelastic second-order analysis

ABSTRACT

Finite elements based on the Euler-Bernoulli beam theory differed from elements that consider Timoshenko theory, once the first theory neglects the deformation due to shear and hence it is not suitable for thick and short beams. In Timoshenko beam formulation, the cross-sections remain plane but not perpendicular to the neutral axis after deformation due the effects of shear strains. This paper presents the development of a finite element model to be used in the inelastic second-order analysis of planar steel frames. The finite element model considers the spread of plastification within the cross-section and along the member length, several residual stress distributions, members shear deformations based on the Timoshenko theory, P-Δ and P-δ second order effects. The proposed theoretical development considers the updated Lagrangian formulation and the corotational technique for the consistent deduction of the element tangent stiffness matrix. The theory predicts that nodes will suffer large displacements and rotations, and the elements of the structure, large stretches and curvatures. A computer program capable of performing advanced inelastic analysis is developed and numerical examples are presented in order to prove the efficiency of the proposed formulation. It is shown that the Timoshenko beam model is clearly superior to Euler-Bernoulli model in precisely predicting the structural response.

1. Introduction

The analysis of structural elements behavior is generally carried out using the classical Euler-Bernoulli theory, in which cross-sections remain plane and perpendicular to the neutral axis after deformation. However, this theory does not consider the effects of shear strains in the cross-sections and, therefore, is more appropriate for members that have a relatively high span to depth ratio. In the case of thick and short beams, especially beams with I-shaped cross-section, this theory is not suitable, since the influence of shear deformation is more pronounced due to high shape factors. Consequently, in these cases, the shear effects can not be neglected in the analysis. Based on this fact, numerous efforts have been made in order to create finite element models including shear deformation effects for analysis of beams and frames.

Timoshenko [1] extended the validity limit of the classical theory by introducing the shear effects, constant along the height of the cross-section of the bar, into the differential equations that govern the problem. Thus, in Timoshenko beam formulations, the cross-sections remain plane but not perpendicular to the neutral axis after deformation. Cowper [2] presented a revised formulation of Timoshenko's beam theory starting from the equations of linear elasticity for an isotropic homogeneous beam. It was developed a formula for the shear correction factor based on solutions of a cantilever beam subjected to a tip load, introducing residual displacement terms defined as the difference between the actual displacement in the beam and the average displacement representation. Following Cowper's approach, Stephen [3] computed the shear correction factor for beams of various cross-sections by using the exact solutions in a beam subject to a uniform gravity load. Ghugal and Sharma [4] developed a hyperbolic shear deformation theory for the static flexure analysis of thick homogeneous isotropic beams. The theory does not require the use of shear correction factor once the transverse shear stress can be obtained directly from the use of constitutive relations.

Several finite element models that consider the hypotheses of Timoshenko have already been reported in the literature. The simplest Timoshenko beam element was attributed to Hughes et al. [5], who considered the two-node element with linear approximation for the transverse displacement and cross-section rotation. The finite element model presented reasonable result for beams with low ratio between the length of the bar and the height of the cross-section. However, the

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https://doi.org/10.1016/j.jcsr.2018.09.011

model suffers the shear-locking problem due to the inconsistent order of the interpolation functions used for the transverse displacements and rotations, causing the locking of the solution. Friedman and Kosmatka [6] developed the stiffness, mass, and consistent force matrices for a simple two-node Timoshenko beam element based on Hamilton's principle. Interdependent cubic and quadratic Lagrangian polynomials are used for the transverse and rotational displacements, respectively. Luo [7] developed an efficient tridimensional two-node Timoshenko beam element. The shape functions of the element were defined from homogeneous Euler-Lagrangian equations. Numerical results showed that the developed Timoshenko beam element does not present shear locking problems. Bazoune et al. [8] extended the expressions of shape functions developed by Przemeniecki [9] to a three-dimensional Timoshenko beam element. It was used the conventional cubic Hermitian polynomials for translational and rotational bending deformation, which include, in addition to the completeness and continuity conditions, the shear deformation factors that account for shear effects. Linear shape functions are considered for torsional and axial deformation.

Although the literature presents plenty of other interesting nonlinear analysis formulations based on Timoshenko theory, there are still a lack of methods that consider the shear deformation effects associated with the important attributes that affects the behavior of steel frames structures, such as the geometric and material nonlinearities. The geometric nonlinearity includes second-order effects associated with the P-δ and P-Δ effects and geometric imperfections. The material nonlinearity includes spread of yielding or plasticity associated with the influence of residual stresses. A realistic modeling of a steel frame requires the use of these attributes if an accurate response of the frame is desired. The effect of shear deformation can contribute significantly to the nonlinear behavior of structures. Consequently, the Timoshenko's theory has been extensively investigated and applied in nonlinear analysis.

In recent decades, research efforts have been devoted to the development and validation of several nonlinear inelastic analysis methods for steel frames with rigid and semi-rigid connections, as the studies presented by Foley e Vinnakota [10,11], Iu and Bradford [12], Saritas and Koseoglu [13], Nguyen and Kim [14,15], Rezaiee-Pajand and Gharaei-Moghaddam [16], Nimnim and Mahdi [17], Da Silva and Silva [18], among other researchers. With the advance of research in computational development, the geometric and material nonlinearities have been progressively incorporated into the analysis, resulting in a more realistic global structure response.

Nguyen and Kim [14] presented a second-order distributed plasticity analysis method for steel frames using only one beam-column element per frame member by employing stability functions, in which the elemental axial stiffness and bending stiffness are sensitive to the number of integration points due to the important influence of weight factors. To obtain a high accuracy, Nguyen and Kim [15] presented a new numerical procedure for analysis of planar steel frames under static loadings in which the second-order effects and distributed plasticity was considered by dividing the member into several sub-elements and meshing the cross-section into several fibers.

Rezaiee-Pajand and Gharaei-Moghaddam [16] proposed a 2D Timoshenko beam element, which was formulated by using variational principles and force method. The 2D Timoshenko frame element takes into account material and geometrical nonlinearities. Material nonlinearities were considered by fiber discretization technique. In addition, it was considered axial force and bending moment interaction. As the corotational formulation was employed, this element can be used in problems with large rigid body rotations and displacements. The formulation is shear locking-free and numerical results proved that the proposed element provides acceptable precision for practical frame problems.

Da Silva and Silva [18] evaluated the efficiency of a 2D Timoshenko beam element applied in a finite element program that considers the geometric and material nonlinearities. A corotational formulation and a layered plastic modeling based on the rate-independent Von-Mises criterion with isotropic hardening were employed.

Recently, in order to predict more accurately the mechanical responses of functionally graded material (FGM) plates, various studies on beams based on new Quasi-3D shear deformation theory have been reported in the literature, as the works presented by Thai et al. [19], Hebali et al. [20] and Neves et al. [21]. Additionally, some research works have been published using new shear deformation theories for advanced composite structures to examine the, mechanical, thermal and thermo-mechanical behavior of plates, as the studies presented by Houari et al. [22], Hamidi et al. [23], Bouderba et al. [24], Khetir et al. [25] and Abualnour et al. [26].

This paper presents the development of an exact geometrically nonlinear formulation to be used in the inelastic second-order analysis of planar steel frames including shear deformations based on the Timoshenko theory. Theoretical development takes into account the updated Lagrangian formulation and the corotational technique for the consistent deduction of the constitutive and geometric tangent stiffness matrix of the element with both fixed ends. The theory predicts that nodes will suffer large displacements and rotations, and the elements of the structure, large stretches and curvatures. Furthermore, these elements may be nonhomogeneous and nonprismatic. The finite element model considers the spread of plastification within the cross-section and along the member length, several residual stress distributions, P-δ and P-Δ second order effects. The transverse displacement including bending and shear deformation has its field interpolated by a cubic polynomial, while the cross-section rotation is interpolated by a quadratic polynomial. Due to the consistency of the order of the employed interpolation functions, the finite element model is free of shear locking. As a result, it is presented a new tangent stiffness matrix that incorporates the shear deformation effects.

The advantage of the proposed methodology is that it can sufficiently capture the limit state strength and stability of the structural system and its individual members so that separate member capacity checks are not required. The limit load obtained by the proposed formulation is closer to the real load resistance if compared, for example, to the one obtained by a second-order elastic analysis via the Euler-Bernoulli theory. Moreover, the present formulation is suitable for analysis of thick and short beams since the shear deformation effects are not neglected.

A computer program associated with the finite element model is developed. Numerical examples are presented to demonstrate the efficiency of the formulation in the solution of problems that consider geometric and material nonlinearities and highlight the effects of the shear deformations, especially in the case in which the length-to-height ratio of the bar is small.

2. Finite element model

The 2D frame element with both fixed ends is shown in Fig. 1. The structural nodes have three degrees of freedom, namely, the displacements u and v along the axis x and y, respectively, and the rotation θ, positive when counter-clockwise. In the reference configuration, the chord between elements nodes has the length $\mathbf{l_r}$. On the chord a local reference coordinate system (x_r, y_r) is placed, with the origin at the center. The angle between the axis **x** and the chord is denoted by $\phi_\mathbf{r}$. At current configuration, the chord between element nodes has length $\mathbf{l_c}$. A corotational coordinate system (x_c, y_c) is defined on this chord, with the origin at the center, as indicate in Fig. 1. The angle between the axis x and the chord is now $\phi_\mathbf{c}$ while the angle between the chord and the axis of the bar is denoted by α.

The natural and Cartesian degrees of freedom of the element are defined, respectively, by:

$$q_\alpha^T = \{q_1 = l_c - l_r;\ \ q_2 = \alpha_a;\ \ q_3 = \alpha_b\}; p = \{u_a; v_a; \theta_a; u_b; v_b; \theta_b\} \tag{1}$$