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Numerical research on elasto-plastic behaviors of fiber-reinforced polymer based composite laminates

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ABSTRACT

A new pressure-dependent elasto-plastic constitutive model for polymer based composite laminates with unidirectional plies is established and implemented into finite element software to simulate off-axis loading tests of two kinds of composites. The validity of the new plasticity model is proved by available off-axis loading test results. Additionally, accuracies and efficiencies of the backward Euler algorithm and the forward Euler algorithm respective in solving plastic deformation of fiber-reinforced composite materials are analyzed. Compared with the forward Euler algorithm based plasticity model, the backward Euler scheme shows comparative prediction accuracy of laminate plastic behavior and better computational efficiency. This paper is expected to facilitate the numerical predictions of composite laminates' elasto-plastic behaviors for researchers and engineers.

1. Introduction

In order to be applied in aerospace, military, automotive, civil and marine industries, engineering designed fiber-reinforced polymer matrix composite structures are minimally required to possess satisfactory load capacities, impact resistances as well as damage tolerances. Apart from experimental tests, acquirement of those features mainly relies on mechanical analysis of composite structures' responses under service loading conditions, where precise predictions of composite material's failure process are essential. The pre-failure stresses and strains in materials are necessities for predicting failure occurrence and damage status of composites. Therefore, accurate descriptions of composite material constitutive behaviors under complex loading conditions are significant.

Prior to failure, nonlinear macroscopic stress-strain relationships and unrecoverable deformations were observed in off-axis loading experiments [1] and cyclic tensile tests [2–4] of fiber-reinforced composite laminates. Those nonlinearities can be attributed to material plasticity. A number of elasto-plastic constitutive models consisting of different yielding functions, flow rules and numerical implementation algorithms have already been established, verified and utilized to analyze composite materials' plastic behaviors under diverse loading conditions.

The one parameter 2-D plasticity model proposed by Sun and Chen [5] and the corresponding 3D extension formulated by Week and Sun [6] are commonly seen utilized by researchers to predict plasticity of fiber-reinforced composite materials in the ply level. An isotropic hardening law showing the power function form and the associated flow rule were used in their formulations. This kind of one-parameter based plasticity models proved to be capable of capturing unidirectional and multidirectional composite laminates' plastic behaviors in tensile tests [7–9] and predicting laminate inelastic deformations in transverse low velocity impact events despite of minor discrepancies between predictions and test results [10,11].

Although the one parameter plasticity models could well predict polymer-based laminate plastic responses, the pressure-sensitivity of material yielding [12] was not well represented. Coping with this occasion, pressure-dependent plasticity model with non-associated flow rule was developed. Based on Sun's work [6], Yokozeki et al. [13] proposed a two-parameter plasticity model and incorporated the hydrostatic-pressure term in their yielding function to consider the pressure sensitivity. Meanwhile, the influence of stress along fiber direction on material plasticity confirmed by Frans [14] and Mena-Tun et al. [15] was also considered in this model. According to the study of Higuchi et al. [16], combined with appropriate damage mechanics based failure model, the two-parameter plasticity model could accurately predict the nonlinear progressive damage of laminates. In addition, a invariant theory based yield criterion was formulated by Vogler et al. [17] and all of the predicted nonlinear stress-strain curves agreed well with several

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uniaxial compression tests under different levels of hydrostatic pressures. Vyas et al. [18] and Singh et al. [19] took advantages of Raghava's criterion [20] and also well captured composite pressure-dependent plasticity respectively in multi-axial loading and transverse low velocity impact.

Within above mentioned plasticity models, appropriated numerical implementation algorithms are essentially necessary for numerically implementing the constitutive model of composite material's elasto-plasticity. Applied in finite element analysis, two kinds of strain-driven procedures updating plastic deformations respectively based on forward Euler algorithm and backward Euler algorithm are commonly utilized by researchers, both of which exhibit satisfactory performances in finite element analysis. Based on forward Euler algorithm, researchers [10,13,18,21,22] updated plastic multiplier by solving consistency equation and acquired the explicit form of plastic incremental strains based on trial state variables and plastic responses of composites observed in experimental tests could be well regenerated via the forward Euler algorithm. Hoffarth et al. [23] utilized secant iteration strategy in the forward Euler procedure to keep updated stress-strains staying on the yielding surface. When backward Euler integration algorithm is utilized, the plastic multiplier is mostly computed by solving the nonlinear yielding equation within Newton iterations and finally converged plastic strains are obtained. Chen et al. [8], Vogler et al. [17], Higuchi et al. [16] all found that predictions of composite plasticity based on backward Euler algorithm made good correspondence with test results.

Although the computed stress state cannot well return back to the yield surface via forward Euler algorithm just like that does by backward Euler algorithm, the predicted plastic deformation of laminates subjected to transversely low velocity impact by both algorithms were nearly the same in Liao and Liu's study [11]. Whereas, due to the influence of damage occurrence in the study, accuracies of the backward Euler algorithm-based plasticity model and the forward Euler algorithm-based plasticity model respective in solving plastic deformation of fiber-reinforced composite materials have not been well clarified. Meanwhile, comparisons of these two kinds of plasticity models' capabilities are rarely documented in literatures in the best of the authors' knowledge. In order to facilitate engineers' and researchers' satisfactory selection between the forward Euler based constitutive plastic model and the backward Euler based constitutive plastic model, two questions respectively to what extent both models could predict fiber-reinforced composite materials' plastic behaviors in loading conditions and which one performs more accurately and efficiently have to be resolved.

This paper builds a new constitutive pressure-dependent plastic model within the non-associative flow rule based on modified two-parameters' plastic yielding function originally proposed by Yokozeki et al. [13] and implements the model respective by forward Euler algorithm and backward Euler algorithm into finite element analysis. Two cases' off-axis compression tests of composite laminate's responses are simulated. The analyzed results are examined against test results in literatures [6,24]. In this paper, Section 1 presents the literature of constitutive modeling of composite material's plasticity and states the research focus. Section 2 constructs a new pressure-dependent plasticity model for polymer-based composites and elaborates on the numerical procedures respectively based on forward Euler algorithm and backward Euler algorithm. The finite element modeling of off-axis compression test is presented in Section 3. Discussions and comparisons are made in Section 4 for verifying the proposed plasticity model and analyzing two different solution algorithms. Conclusions are drawn in Section 5.

2. Constitutive description for elasto-plastic laminate ply

2.1. Mathematical formulation

Focusing on polymer-based composites' purely elasto-plastic constitutive behavior, this article neglects the thermal effects, material rate-dependence and curing residual stresses in composites. The initial orthotropic elasticity of homogenized ply is assumed and the relationship between elastic stress and strains is formulated as

$$\boldsymbol{\sigma} = \boldsymbol{C}\cdot\boldsymbol{\varepsilon}^{\mathrm{e}} = \mathbf{C}\cdot(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathrm{p}}) \tag{1}$$

$$\boldsymbol{C} = \frac{1}{\Omega}\begin{bmatrix} E_1(1-\nu_{23}\nu_{32}) & E_2(\nu_{12}+\nu_{13}\nu_{32}) & E_3(\nu_{13}+\nu_{12}\nu_{23}) & 0 & 0 & 0 \\ E_1(\nu_{21}+\nu_{31}\nu_{23}) & E_2(1-\nu_{13}\nu_{31}) & E_3(\nu_{23}+\nu_{21}\nu_{13}) & 0 & 0 & 0 \\ E_1(\nu_{31}+\nu_{21}\nu_{32}) & E_2(\nu_{32}+\nu_{12}\nu_{31}) & E_3(1-\nu_{12}\nu_{21}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Omega G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Omega G_{31} \end{bmatrix} \tag{2}$$

where is $\Omega = 1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{13}\nu_{31}-2\nu_{12}\nu_{31}\nu_{23}\cdot\boldsymbol{\sigma} = [\sigma_{11}\ \ \sigma_{22}\ \ \sigma_{33}\ \ \sigma_{12}\ \ \sigma_{23}\ \ \sigma_{13}]^{\mathrm{T}}$ $\boldsymbol{\sigma} = [\sigma_{11}\ \ \sigma_{22}\ \ \sigma_{33}\ \ \sigma_{12}\ \ \sigma_{23}\ \ \sigma_{13}]^{\boldsymbol{T}}$ the elastic stress and $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^{\mathrm{e}}$, $\boldsymbol{\varepsilon}^{\mathrm{p}}$ are respectively total strains, elastic strains and plastic strains. E_1, E_2, E_3 are respectively young's modulus along fiber direction and in-plane and out-of-plane transverse directions. G_{12}, G_{23}, G_{31} are shear moduli and ν_{ij} ($i \neq j$ and i, j = 1, 2, 3) are Poisson ratios.

With the consideration of the influence of hydrostatic pressure on polymer-matrix composites' mechanical responses [17,18,25], the basic form of pressure-dependent effective stress defined by Yokozeki et al. [13] is utilized here. A modification is made to the formulation of Yokozeki's plastic potential to incorporate the hydrostatic pressure term into the square root function, which way leads to a simplified form of incremental plastic strain based on the non-associative flow rule rather than the complicated one shown in the Appendix A. Therefore, a new pressure-dependent plastic potential function as well as a new formulation of effective stress is obtained as Eqs. (3) and (4), in which a_{11}, a_{44}, a_{66} denote coefficients indicating the anisotropy in plasticity [5] and are commonly obtained via handling the off-axis loading test results.

$$g = \frac{1}{2}[(\sigma_{22}-\sigma_{33})^2 + 2a_{66}(\sigma_{12}^2 + \sigma_{13}^2) + 2a_{44}\sigma_{23}^2] + \frac{1}{3}a_{11}^2(\sigma_{11} + \sigma_{22} + \sigma_{33})^2 \tag{3}$$

$$\widetilde{\sigma}_{\mathrm{eff}} = \sqrt{\frac{3}{2}[(\sigma_{22}-\sigma_{33})^2 + 2a_{66}(\sigma_{12}^2 + \sigma_{13}^2) + 2a_{44}\sigma_{23}^2] + a_{11}^2(\sigma_{11} + \sigma_{22} + \sigma_{33})^2} \tag{4}$$

Consequently, the pressure-dependent yielding function is defined as

$$F(\boldsymbol{\sigma}, \varepsilon_{\mathrm{eq}}^{\mathrm{p}}) = \sqrt{3\cdot g} - R(\varepsilon_{\mathrm{eq}}^{\mathrm{p}}) \tag{5}$$

where $R(\varepsilon_{\mathrm{eq}}^{\mathrm{p}})$ represents the isotropic hardening characteristics. The following exponential function is adopted to describe isotropic hardening, in which R_0, A_i, B_i (i = 1, 2, 3) are parameters produced by fitting the nonlinear plastic hardening response.

$$R(\varepsilon_{\mathrm{eq}}^{\mathrm{p}}) = R_0 + A_1\cdot e^{-\frac{\varepsilon_{\mathrm{eq}}^{\mathrm{p}}}{B_1}} + A_2\cdot e^{-\frac{\varepsilon_{\mathrm{eq}}^{\mathrm{p}}}{B_2}} + A_3\cdot e^{-\frac{\varepsilon_{\mathrm{eq}}^{\mathrm{p}}}{B_3}} \tag{6}$$

According to the study of Vogler et al. [17], the non-associated flow rule outweighs the associated flow rule in terms of predicting realistic plastic Poisson coefficients and volumetric plastic strains. The non-associated flow rule is utilized here to acquire the incremental plastic strains $\boldsymbol{\varepsilon}^{\boldsymbol{p}}$ and its formulation is as Eq. (7), in which $d\lambda$ is the plastic multiplier.