Composite Structures 207 (2019) 560–575

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A modified Fourier solution for sound-vibration analysis for composite laminated thin sector plate-cavity coupled system

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ARTICLE INFO

Keywords:
Plate-cavity coupled system
Laminated thin sector plate
Sound-vibration characteristics
Rayleigh-Ritz technique
Weakly coupled system
Strongly coupled system

ABSTRACT

This paper applied the modified Fourier series method to investigate the sound-vibration characteristics by establishing a composite laminated thin sector plate-cavity coupled model for the first time based on the classical plate theory (CPT) and Rayleigh-Ritz energy technique. The coupled system consists of an annular sector or circular sector plate backed by an acoustic cavity filled with air or water. Ignoring the influence of boundary conditions, displacements admissible functions of laminated sector plate and sound pressure admissible functions of cavity can be set up as a Fourier series superposition, whose composition are the superposition of Fourier cosine series and supplementary functions. The addition of these supplementary polynomials can effectively eliminate the discontinuity or jump phenomenon on the boundary. The correctness of the established analytical model has been validated by being compared with the results achieved by the finite element method (FEM). On this basis, the coupling mechanism of the weakly coupled system and the strongly coupled system are discussed in detail. In addition, some new results and discussions are given, including the cavity depth, plate thickness, anisotropic degree, varying boundary conditions and so on, which could provide reference for future research.

1. Introduction

A coupled fluid–structure system consisted of an elastic plate and an acoustic cavity is an abstract model for many engineering applications. The research on the interaction between the elastic plate and acoustic cavity is of great guiding significance and practical application value. So far, the study on rectangular composite plate-cavity coupling system [1–8] is relatively extensive. However, the research on the laminated sector plate-cavity coupled system is obviously insufficient. In view of the wide application of the sector coupled system in aerospace, ship, automobile and subway, it is very important to establish an efficient and reliable analysis model to analyze the sound-vibration characteristics.

In recent years, the study of annular, circular and sector plates has attracted the attention of many researchers. On the basis of the first order shear deformation theory (FSDT), Jin et al. [9] developed a unified modified Fourier method to discuss the vibration problems of varying laminated structure elements of revolution such as annular plates. Then, they [10] investigated the effects of the material, geometrical properties and boundary constraints on free vibration of the composite functionally graded and laminated sector-like plates. In addition, they [11] examined the vibration problems of the moderately thick composite laminated annular sector plate with varying supports, such as the general uniform, circumferential arc and internal radial line supports. Sharma et al. [12] first proposed the free vibration results of laminated sector-like plate on the basis of a differential quadrature method (DQM). In this paper, the elastic edges of the laminated sector plates were realized by varying the edge stiffness. Powmya et al. [13] presented the solutions for the axisymmetric vibration analysis of polar orthotropic or laminated circular and annular plates according to collocating principle of the motion equations at Chebyshev zeroes. Khare et al. [14] presented a numerical solution for the free vibration of laminated thick circular plates by using FEM and three-dimensional theory. Mercan *et al.* [15,16] studied the vibration problem of orthotropic composite annular plate on the basis of FSDT and discrete singular convolution (DSC). In fact, we [17–20] also studied the vibration problems of the orthotropic and laminated composite annular or sector plate under elastic uniform boundary conditions circumferential arc and internal radial line supports. Although these studies have not extended from the structure domain to the acoustic domain, they provide a solid foundation for the following research on the structure-acoustic coupled system.

There have been some literatures for the rotational structural–acoustic coupling systems including both the structure domain and

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https://doi.org/10.1016/j.compstruct.2018.09.037
Received 16 August 2018; Accepted 17 September 2018

the sound field. Li et al. [21] proposed a method of Radiation Efficiency Analysis of Structural Modes to study the sound-vibration coupled characteristics of the cylindrical shell coupled with internal floor partition. Henry et al. [22,23] established a model composed of a typical aircraft plate coupled with a rigid-wall cylinder with attached piezoelectric actuators in service conditions to realize the active control of sound transmission. Gardonio et al. [24] investigated the plane wave transmission characteristics of the circular cylindrical sandwich shell in the aerospace industry for satellite launch vehicles based on a modal interaction analysis (MIA). Lee et al. [25] examined the insertion loss of a cavity-backed semi-cylindrical enclosure panel, and the theoretical results were validated by the experimental results. According to a circularly metamaterial model, Yao et al. [26] analyzed the sound radiation properties based on the numerical simulations for structured metamaterials. It was found that it is an efficient method to reduce the sound vibration over the negative-mass frequencies with increasing the thickness of the metamaterial barrier. Rocha et al. [27] established a structural–acoustic coupling system composed of aircraft cabin section and the fuselage structure to predict turbulent boundary-layer-induced noise in the interior of aircraft cylindrical cabins. From these researches, we can find that there are few studies on the interaction and coupling mechanism of the structure-acoustic coupled system. And most of the coupling system is weakly coupled system with air as medium rather than the strongly coupled system with water as medium. In fact, to the best of the authors' knowledge, no such studies are available in the open literature that discusses the sound-vibration characteristics of the composite laminated thin sector plate-cavity coupled system filled air or water.

In view of the limitations of existing research, this paper establishes a composite laminated thin sector plate-cavity coupled system to investigate the sound-vibration characteristics according to the modified Fourier series method and CPT. The coupled system could be annular sector or circular sector which consists of an elastic plate backed by an acoustic cavity filled with air or water. The correctness of the established analytical model has been validated by being compared with the results achieved by FEM. On this basis, the coupling mechanism of the weakly coupled system and the strongly coupled system are discussed in detail. In addition, some new results and discussions are given, including the cavity depth, plate thickness, anisotropic degree, varying boundary conditions and so on, which could provide reference for future research.

2. Theoretical formulations

2.1. Modeling of the sector plate-cavity coupled system

As mentioned earlier, this paper focuses on the sound-vibration characteristics of the composite laminated thin annular sector plate-cavity coupled system and circular sector plate-cavity coupled system, which can be collectively referred to as rotary fluid–structure coupled system. As we all know, the circular sector plate can be regarded as a special case of the annular sector plates. The circular sector plate structure can be obtained only if the inner radius of the annular sector plate is set to zero. Therefore, in this model, as long as the parameter R_0 is modified, both of these two models can be studied at the same time. The geometric size and coordinate system of the sector fluid–structure coupled systems are given in Fig. 1. As shown in Fig. 1(a), the geometric dimensions include inner radius (R_0), outer radius (R_1), sector angle (ϕ), plate thickness (h_p) and cavity depth (h_c). In addition, the difference of inner radius and outer radius is marked as R ($R = R_1 - R_0$). The whole coordinate system adopts a cylindrical coordinate system (*o-r, θ, z*) which is established in the middle surface of laminated thin plate. The intervals of *r, θ,* and *z* are: $R_0 \leqslant r \leqslant R_1$, $0 \leqslant \theta \leqslant \phi$, and $(-h_p/2) \leqslant r \leqslant (h_c + h_p/2)$, respectively. However, for the annular sector plate-cavity coupled system, it is necessary to establish the local coordinate system which is (*o-s, θ, z*), whose purpose is to make the left interval value of the radius direction to zero. It is easy to see that the relationship of r and s is: $s = r - R_0$. In addition, the laying angle between the layer fiber direction and r-axis of the laminated thin plate is marked as the symbol α. According to the artificial virtual spring technology [28–30] and the displacement function of the thin sector plate, the four sets of springs (k_u, k_v, k_w and K_w) are evenly distributed on the plate's edges to realize the various elastic supports. Different boundary constraints can be easily obtained by setting four sets of spring stiffness values on the boundaries. The point force F placed on the plate's surface or the monopole pointe sound source Q placed inside the cavity are applied to study the sound-vibration responses.

2.2. Admissible functions of coupled system

Take the laminated thin annular sector plate-cavity coupled system as an example. According to the classical thin plate theory (CPT), the overall displacements of the annular sector plate could be established based on the displacements of the laminated plate's mid-surface. Their specific expressions could be written as:

$$
\begin{aligned}
U(s,\theta,z,t) &= u(s,\theta,t) - z_p\frac{\partial w(s,\theta,t)}{\partial r}\\
V(s,\theta,z,t) &= v(s,\theta,t) - z_p\frac{\partial w(s,\theta,t)}{(s+R_0)\partial\theta}\\
W(s,\theta,z,t) &= w(s,\theta,t)
\end{aligned}
\tag{1}
$$

in which u, v and w donate displacements of the laminated thin annular sector plate on the mid-surface in s, θ and z directions, separately. Besides, z_p is the thickness variable of the laminated annular sector plate, and t is the time variable.

In this paper, a modified Fourier series method is extended to investigate the sound-vibration characteristics of the laminated thin annular sector plate or circular sector plate [31] coupled with the acoustic cavity, which have different elastic supports and varying impedance-walls. For the laminated thin plate, the two-dimensional (2D) Fourier series expressions [32,33] are used to indicate the plate's displacements. At the same time, the three-dimensional (3D) Fourier series expression is used to indicate the sound pressure of the cavity. These expressions can be expressed as superposition of the product of cosine functions and the supplementary polynomials. The introduction of supplementary polynomials can keep the continuity or remove the jump phenomena on the boundaries or the walls, and facilitate the study of elastic supports and impedance walls. The specific displacement expressions and sound pressure expressions of the coupled system can be written as follow:

$$u(s,\theta,t) = e^{-j\omega t}\left(\boldsymbol{U}^{\Omega}(s,\theta) + \sum_{q=1}^{2}\boldsymbol{U}_q^{\mathrm{B}}(s,\theta)\right)\boldsymbol{A}_{mn} \tag{2}$$

$$v(s,\theta,t) = e^{-j\omega t}\left(\boldsymbol{V}^{\Omega}(s,\theta) + \sum_{q=1}^{2}\boldsymbol{V}_q^{\mathrm{B}}(s,\theta)\right)\boldsymbol{B}_{mn} \tag{3}$$

$$w(s,\theta,t) = e^{-j\omega t}\left(\boldsymbol{W}^{\Omega}(s,\theta) + \sum_{q=1}^{2}\boldsymbol{W}_q^{\mathrm{B}}(s,\theta)\right)\boldsymbol{C}_{mn} \tag{4}$$

$$p(s,\theta,z,t) = e^{-j\omega t}\left(\boldsymbol{P}^{\Omega}(s,\theta,z) + \sum_{i=1}^{6}\boldsymbol{P}_i^{\mathrm{S}}(s,\theta,z)\right)\boldsymbol{D}_{m_1n_1l_1} \tag{5}$$

in which $\mathbf{U}^{\Omega}$, $\mathbf{V}^{\Omega}$, $\mathbf{W}^{\Omega}$ and $\mathbf{P}^{\Omega}$ are the internal displacements and sound pressure distribution function for the fluid–structure coupled system. ${\mathbf{U}_q}^{\mathrm{B}}$, ${\mathbf{V}_q}^{\mathrm{B}}$, ${\mathbf{W}_q}^{\mathrm{B}}$ ($q = 1$ and 2) and ${\mathbf{P}_i}^{\mathrm{S}}$ ($i = 1$–6) express the supplementary polynomials on the boundaries and walls. In addition, $\mathbf{A}_{mn}$, $\mathbf{B}_{mn}$, $\mathbf{C}_{mn}$ and $\boldsymbol{D}_{m_1n_1l_1}$ are unknown Fourier coefficients vector, separately. They are made up of the 2D Fourier coefficients A_{mn}^e, B_{mn}^e, C_{mn}^e ($e = 1$, 2 and 3) and 3D Fourier coefficients $D_{m_1n_1l_1}^f$ ($f = 1$–7), respectively.