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Prediction of *in situ* strengths in composites: Some considerations

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ABSTRACT

The classic formulation that relates the fracture toughness with the *in situ* strength of the ply implicitly assumes that a fracture process zone fully develops within the ply. This assumption, reasonable for conventional composite laminates, may not be appropriate when the ply thickness is very small or the fracture process zone very large. In the following it is shown how considering the R-curve of the material, the *in situ* strength for the cases when the fracture process zone cannot develop completely can be correctly computed. Closed form solutions are found for the *in situ* strengths, and for their maximum values that are obtained when the ply thickness approaches zero.

1. Introduction

Strength prediction methods of composite structures rely on the ability of failure criteria to predict the ply damage onset associated with a given failure mechanism. When the ply is embedded in a laminate, phenomenological failure criteria [1–7] are usually written in terms of the *in situ* strengths, i.e. the strengths exhibited by the ply when embedded in a laminate. It is well known that when a ply is part of a multidirectional laminate it exhibits a transverse strength that is usually higher than the strength of the same ply when part of a unidirectional laminate [8]; this effect is usually referred as *in situ* effect. The *in situ* effect depends, in a first approximation, on the thickness of the ply [9], and in a second approximation, on other parameters as the lay-up of the laminate and the manufacturing process.

An analytical model to predict the *in situ* strengths was proposed by Camanho et al. [9] for thick and thin plies under transverse tensile, Y_T^{is}, and in-plane shear loading, S_L^{is}. Later, it was also shown that the transverse strength in compression, Y_C^{is}, and the transverse shear strength, S_T^{is}, are *in situ* parameters that can be estimated if the *in situ* strength in shear, S_L^{is}, is known [6]. All these models have been intensively used, and the ability to correctly estimate the strength of the material have been proved for conventional laminates (e.g. laminates manufactured using a thermosetting resin with a thickness in the range of 0.1 to 0.2 mm).

New material systems have however challenged the ability of the classic formulation to predict the *in situ* strength. For thermoplastic composites [10], and for (ultra)-*thin-ply* laminates [11,12], the use of the classic formulation may provide a substantial overestimation of the *in situ* strengths.

Considering a constant value of the fracture toughness is more than reasonable for conventional composites, but leads to well known pathological issues: first of all, the *in situ* strengths asymptotically tend to infinity when the thickness approaches zero. The following shows how taking into account the R-curve of the material could mitigate and/or eliminate all these issues, and could lead to a correct estimation of the *in situ* strengths.

2. Analytical model

2.1. In situ tensile strength of a thin embedded ply

The *in situ* tensile strength of a thin embedded ply, Y_T^{is}, can be obtained solving the following Equation [9,13]:

$$\mathscr{G}_{Ic} = \frac{\pi t}{8}\Lambda_{22}^{o}\,Y_T^{is\,2} \tag{1}$$

where t is the thickness of the ply, $\mathscr{G}_{Ic}$ is the composite transverse fracture toughness in mode I, and Λ_{22}^{o} is a parameter defined as [13,14]:

$$\Lambda_{22}^{o} = 2\left(\frac{1}{E_2}-\frac{\nu_{12}^2}{E_1}\right) = \frac{2}{E_2}(1-\nu_{12}\nu_{21}) \tag{2}$$

being E_1 and E_2 the ply Young's moduli in the longitudinal and transverse directions, and ν_{12} and ν_{21} the major and minor Poisson's ratios, respectively.

For a very thin ply, or for a thermoplastic composite for which the resin exhibits an R-curve characterized by a large fracture of process zone, it may happen that $l_{fpz} \geqslant t/2$. In this case, Eq. (1) is not appropriate because it assumes that the slit crack of Fig. 1 propagates along the transverse direction, developing a full fracture process zone.

In this case, the R-curve of the material should be considered; it can be expressed in analytical form as [15]:

$$\mathscr{R} = \begin{cases} \mathscr{R}_{ss}(1-(1-\Delta a/l_{fpz})^{\kappa}) & \text{if} \quad \Delta a \leqslant l_{fpz} \\ \mathscr{R}_{ss} & \text{if} \quad \Delta a > l_{fpz} \end{cases} \tag{3}$$

where Δa is the crack increment, $\mathscr{R}_{ss}$ is the steady state value of the R-curve, and κ is a dimensionless parameter. Eq. (3) can be expressed in a more compact form as:

$$\mathscr{R}_{\Delta a} = \mathscr{R}_{ss}\left(\left(1-\left(1-\frac{\Delta a}{l_{fpz}}\right)^{\kappa}\right)H[l_{fpz}-\Delta a] + H[\Delta a-l_{fpz}]\right) \tag{4}$$

where $H[x]$ is the Heaviside step function defined as:

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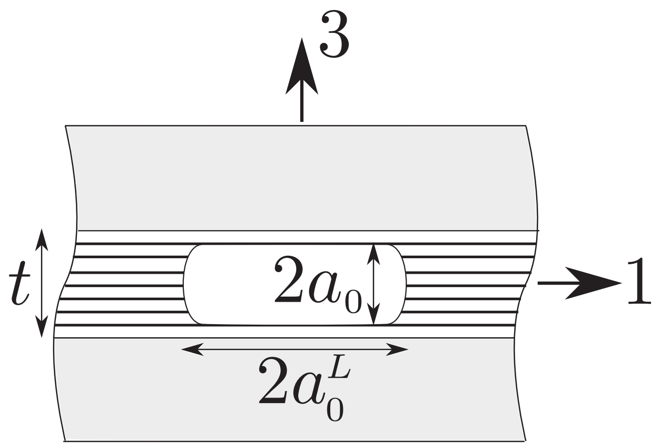

Fig. 1. Slit crack (after [13]).

$$H[x] = \begin{cases} 1 & \text{if} \quad x > 0 \\ 1/2 & \text{if} \quad x = 0 \\ 0 & \text{if} \quad x < 0 \end{cases} \tag{5}$$

Replacing the fracture toughness in Eq. (1) with the R-curve of Eq. (4) yields:

$$\bar{\mathscr{R}}_{t/2} = \bar{\mathscr{R}}_{ss}\left(\left(1-\left(1-\frac{t/2}{\bar{l}_{fpz}}\right)^{\bar{\kappa}}\right)H\left[\bar{l}_{fpz}-\frac{t}{2}\right] + H\left[\frac{t}{2}-\bar{l}_{fpz}\right]\right) = \frac{\pi t}{8}\Lambda_{22}^{o}\,Y_T^{is2} \tag{6}$$

where the bar accent denotes any parameter that defines the mode I intralaminar R-curve. Eq. (6) can be solved for the *in situ* tensile strength, Y_T^{is}:

$$Y_T^{is} = \sqrt{\frac{8\,\bar{\mathscr{R}}_{t/2}}{\pi\, t\, \Lambda_{22}^{o}}} \tag{7}$$

The maximum value of the *in situ* strength, $\hat{Y}_T^{is}$, can be calculated as the limit of (7), when t approaches 0, as:

$$\hat{Y}_T^{is} = \lim_{t\to 0}\sqrt{\frac{8\,\bar{\mathscr{R}}_{t/2}}{\pi\, t\, \Lambda_{22}^{o}}} = \lim_{t\to 0}\sqrt{\frac{8\,\bar{\mathscr{R}}_{ss}\left(1-\left(1-\frac{t}{2\bar{l}_{fpz}}\right)^{\bar{\kappa}}\right)}{\pi\, t\, \Lambda_{22}^{o}}} \tag{8}$$

which is an indeterminate form. However, noting that the Maclaurin expansion of $\left(1-\frac{t}{2\bar{l}_{fpz}}\right)^{\bar{\kappa}}$ reads:

$$\left.\left(1-\frac{t}{2\bar{l}_{fpz}}\right)^{\bar{\kappa}}\right|_{t=0} = 1-\frac{\bar{\kappa}}{2\bar{l}_{fpz}}t + o[t] \tag{9}$$

where $o[t]$ denotes a *little-o* of t, the limit in (8) can easily be calculated as:

$$\hat{Y}_T^{is} = \sqrt{\frac{4\,\bar{\kappa}\,\bar{\mathscr{R}}_{ss}}{\pi\,\bar{l}_{fpz}\,\Lambda_{22}^{o}}} \tag{10}$$

2.2. In situ shear strength of a thin embedded ply

The *in situ* shear strength for a thin embedded ply can be calculated as the real root of [9,13]:

$$\frac{S_L^{is2}}{8\,G_{12}} + \frac{3}{16}\beta S_L^{is4} = \frac{\mathscr{G}_{IIc}}{\pi\, t} \tag{11}$$

where G_{12} and $\mathscr{G}_{IIc}$ are the ply shear in-plane modulus and the composite transverse fracture toughness in mode II, respectively; β is the parameter used by Hahn and Tsai [16] to approximate the non-linear behaviour of the ply in shear:

$$\gamma_{12} = \frac{1}{G_{12}}\tau_{12} + \beta\tau_{12}^{3} \tag{12}$$

where τ_{12} and γ_{12} are the shear stress and strain, respectively. If in Eq. (11) the R-curve is included it follows:

$$\frac{S_L^{is2}}{8\,G_{12}} + \frac{3}{16}\beta S_L^{is4} = \frac{\widetilde{\mathscr{R}}_{t/2}}{\pi\, t} \tag{13}$$

where the tilde accent indicates any material parameter that defines the mode II R-curve (see Eq. (6)). The real root of Eq. (13) reads [9]:

$$S_L^{is} = \sqrt{\frac{(1+\beta\phi G_{12}^{2})^{\frac{1}{2}}-1}{3\beta\, G_{12}}} \tag{14}$$

where:

$$\phi = \frac{48\,\widetilde{\mathscr{R}}_{t/2}}{\pi\, t} \tag{15}$$

The maximum value of the *in situ* shear strength, $\hat{S}_L^{is}$, is calculated as:

$$\hat{S}_L^{is} = \lim_{t\to 0}\sqrt{\frac{(1+\beta\phi G_{12}^{2})^{\frac{1}{2}}-1}{3\beta\, G_{12}}} \tag{16}$$

In an analogous way, as previously done in Eq. (8), the Maclaurin expansion of the function $\left(1-\frac{t}{2\tilde{l}_{fpz}}\right)^{\tilde{\kappa}}$ can be calculated, and the limit of ϕ as t approaches zero reads:

$$\hat{\phi} = \lim_{t\to 0}\phi = \lim_{t\to 0}\frac{48\,\widetilde{\mathscr{R}}_{t/2}}{\pi\, t} = \frac{24\,\widetilde{\mathscr{R}}_{ss}\,\tilde{\kappa}}{\pi\,\tilde{l}_{fpz}} \tag{17}$$

Replacing Eq. (17) into Eq. (16) yields:

$$\hat{S}_L^{is} = \sqrt{\frac{(1+\beta\hat{\phi} G_{12}^{2})^{\frac{1}{2}}-1}{3\beta\, G_{12}}} \tag{18}$$

2.3. In situ strength of a thin outer ply

The formula to be used for the transverse and shear *in situ* strengths for a thin outer ply are the same as reported in [9,13] and will not be reported here for the sake of conciseness. It should be observed that since the surface crack propagates until $a = t$ the fracture toughness should be replaced by the value of the R-curve for $\Delta a = t$, i.e. $\mathscr{R}_t$.

2.4. In situ transverse compressive strength and transverse shear strength

The *in situ* transverse strength in compression, Y_C^{is}, and transverse shear strength, S_T^{is}, can be estimated as [6]: