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Technical Note

Bayesian interpretation of discrete class characteristics

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ABSTRACT

Bayesian interpretation of forensic evidence has become dominated by the likelihood ratio (LR) with a large LR generally considered favourable to the prosecution hypothesis, H_P, over the defence hypothesis, H_D. However, the LR simply quantifies by how much the prior odds ratio of the probability of H_P relative to H_D has been improved by the forensic evidence to provide a posterior ratio. Because the prior ratio is mostly neglected, the posterior ratio is largely unknown, regardless of the LR used to improve it. In fact, we show that the posterior ratio will only favour H_D when LR is at least as large as the number of things that could possibly be the source of that evidence, all being equally able to contribute. This restriction severely limits the value of evidence to the prosecution when only a single, discrete class characteristic is used to match a subset of these things to the evidence. The limitation can be overcome by examining more than one individual characteristic, as long as they are independent of each other, as they are for the genotypes at multiple loci combined for DNA evidence. We present a criterion for determining how many such characteristics are required. Finally, we conclude that a frequentist interpretation is inappropriate as a measure of the strength of forensic evidence precisely because it only estimates the denominator of the LR.

1. Introduction

Bayes theorem is used to measure the relative strengths of two mutually exclusive hypotheses as a result of evidence presented to the court [1–6]. These are generally the prosecution and defence hypotheses (H_P and H_D, respectively). The prosecution hypothesis often asserts that someone or something (a tool, a weapon, a car, for example) is the source of evidence found at a crime scene. The corresponding defence hypothesis asserts that this person or thing is not the source of the evidence. A Bayesian interpretation of these two competing hypotheses involves adjusting belief in their relative truths based on observing how often there have been instances that support them, or as Richard Price noted in the forward to the posthumous publication of Bayes theorem [7]:

Common sense is indeed sufficient to show us that, from the observation of what has in former instances been the consequence of a certain cause or action, one may make a judgement what is likely to be the consequence of it another time and that the larger number of experiments we have to support a conclusion, so much more the reason we have to take it for granted.

It is illustrative to demonstrate the concept using the analogy of a deck of cards. The individual cards (three of hearts, nine of diamonds, king of clubs, *etc*) are analogous to the people or things that could possibly be the source of the evidence. The cards have class characteristics that define a subset of the complete deck. For example, spades constitute one quarter (13/52) of the deck (neglecting jokers) and aces constitute one thirteenth (4/52) of the deck. Hence knowing a class characteristic of a card narrows the pool of possible cards. This has a direct forensic analogy in that, for example, knowing the blood group type of an evidentiary blood sample narrows the pool of potential donors and knowing the colour of a paint chip narrows the pool of potential cars it could be derived from.

Let us then consider a card randomly drawn from a deck. The probability that this card is a spade (S) is given by:

$$P(S) = \frac{13}{52} = \frac{1}{4}$$

The probability that the card is not a spade (the alternative hypothesis) is given by:

$$P\left(\bar{S}\right) = \frac{39}{52} = \frac{3}{4} = 1 - P(S)$$

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It follows that for an ace (A) and an ace of spades (AS):

$$P(A) = \frac{4}{52} = \frac{1}{13} \quad \text{and} \quad P(\overline{A}) = \frac{48}{52} = \frac{12}{13}$$

$$P(AS) = \frac{1}{52} \quad \text{and} \quad P(\overline{AS}) = \frac{51}{52}$$

If $P(AS)$ is the prior or original probability that the card is an ace of spades, then we can update this probability if we know that the card is a spade. The posterior probabilities of AS and $\overline{AS}$ are then:

$$P(AS|S) = \frac{1}{13} \quad \text{and} \quad P(\overline{AS}|S) = \frac{12}{13}$$

Hence we have a higher probability that the card is an ace of spades (and a lower probability that it is not) given that we know it is a spade. If we know that the card is an ace, we can further increase the posterior probability of an ace of spades:

$$P(AS|A) = \frac{1}{4} \quad \text{and} \quad P(\overline{AS}|A) = \frac{3}{4}$$

This situation is analogous to having a better forensic test. For example, we can achieve higher discrimination between people (further narrow the potential donors) if we use DNA profiling rather than blood group typing.

For our card analogy, Bayes theorem tells us that:

$$P(AS|A) \times P(A) = P(A|AS) \times P(AS) \quad \text{or} \quad \frac{1}{4} \times \frac{1}{13} = 1 \times \frac{1}{52} = \frac{1}{52}$$

$$P(\overline{AS}|A) \times P(A) = P(A|\overline{AS}) \times P(\overline{AS}) \text{ or } \frac{3}{4} \times \frac{1}{13} = \frac{3}{51} \times \frac{51}{52} = \frac{3}{52}$$

If we divide these two equalities:

$$\frac{P(AS|A) \times P(A)}{P(\overline{AS}|A)} \times P(A) = \frac{P(A|AS) \times P(AS)}{P(A|\overline{AS})} \times P(\overline{AS})$$

This leads to:

$$\frac{P(AS|A)}{P(\overline{AS}|A)} = \frac{P(A|AS)}{P(A|\overline{AS})} \times \frac{P(AS)}{P(\overline{AS})}$$

This is the form of the familiar expression for Bayesian inference in a forensic setting [8] where evidence, E, is presented to the court and H_P and H_D are mutually exclusive hypotheses, *ie.* $P(H_P) = 1 - P(H_D)$:

$$\frac{P(H_P|E)}{P(H_D|E)} = \frac{P(E|H_P)}{P(E|H_D)} \times \frac{P(H_P)}{P(H_D)} = \text{LR} \times \frac{P(H_P)}{P(H_D)}$$

The likelihood ratio (LR) or “Bayes factor” [9–11] is the ratio of the probabilities of observing the evidence given the prosecution and defence hypotheses. In our card analogy, the LR is dependent on the class characteristic known about the questioned card: the suit (spade, club, diamond or heart) or the rank (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K). We develop this analogy to demonstrate the limited value of a single class characteristic in providing a posterior ratio favourable to the prosecution.

2. Methods

Continuing our card analogy, the LR is the ratio of the probabilities of observing a spade (or an ace) given the alternate hypotheses that the card is or is not the ace of spades. In the case that we know the card is a spade:

$$\frac{P(AS|S)}{P(\overline{AS}|S)} = \frac{P(S|AS)}{P(S|\overline{AS})} \times \frac{P(AS)}{P(\overline{AS})}$$

Substituting numerical probabilities:

$$\frac{1/13}{12/13} = \frac{1}{12/51} \times \frac{1/52}{51/52}$$

$$\frac{1}{12} = 4.25 \times \frac{1}{51}$$

It is 4.25 times more likely that the card is a spade under the hypothesis that it is the ace of spades than under the hypothesis that it is not the ace of spades. Note that we cannot say that it is 4.25 times more likely than not that the card is an ace of spades given that is a spade. It is in fact 12 times less likely. But this is still an improvement on the prior ratio (1/51). For the same reason, we cannot say in court that the prosecution hypothesis is LR times more likely than the defence hypothesis given the evidence. This is the well documented fallacy of the transposed conditional or “prosecutor’s fallacy” [5,12,13]. We can only say that the evidence is LR times more likely under the prosecution hypothesis than under the defence hypothesis.

In the case that we know the card is an ace:

$$\frac{P(AS|A)}{P(\overline{AS}|A)} = \frac{P(A|AS)}{P(A|\overline{AS})} \times \frac{P(AS)}{P(\overline{AS})}$$

$$\frac{1/4}{3/4} = \frac{1}{3/51} \times \frac{1/52}{51/52}$$

$$\frac{1}{3} = 17 \times \frac{1}{51}$$

Both the LR and hence the posterior ratio have increased because our new class characteristic (rank = ace) is more discriminating than our old one (suit = spade). However, the posterior ratio is still less than one. In other words, it is still less likely that the unknown card is an ace of spades than it is not an ace of spades, in spite of a significantly larger LR. However, in a forensic setting, it is the posterior ratio that is of ultimate interest to the court. The problem is that it is never reported because the prior ratio is not generally known. Under what conditions, then, is the posterior ratio likely to favour the prosecution?

3. Results

Let N be the total number of things (people, tools, weapons, cars, *etc*) equally capable of being the source of crime scene evidence. Let n be the number of things (a subset of N) that have the same class characteristic as the evidence. We assume that each of the things is equally likely to have contributed to the evidence and so the prior probabilities of H_P and H_D are:

$$P(H_P) = \frac{1}{N} \quad \text{and} \quad P(H_D) = \frac{N-1}{N}$$

The probability of observing the evidence given the prosecution hypothesis is:

$$P(E|H_P) = 1$$

The probability of observing another thing with the same class characteristic as that observed in the evidence (the defence