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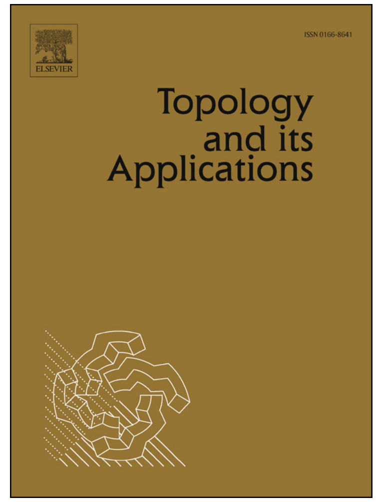

General Bourgin–Yang theorems

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GENERAL BOURGIN–YANG THEOREMS

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ABSTRACT. We describe a unified approach to estimating the dimension of $f^{-1}(A)$ for any G-equivariant map $f\colon X \to Y$ and any closed G-invariant subset $A \subseteq Y$ in terms of connectivity of X and dimension of Y, where G is either a cyclic group of order p^k, a p-torus (p a prime), or a torus.

1. Introduction

The celebrated Borsuk–Ulam theorem [4] states that the existence of a continuous map $f\colon S(\mathbb{R}^n) \to S(\mathbb{R}^m)$ between spheres in Euclidean spaces with the property $f(-x) = -f(x)$ for all $x \in S(\mathbb{R}^n)$ implies that $n \leq m$. Consequently, if $g\colon S(\mathbb{R}^n) \to \mathbb{R}^m$ is a continuous map with that property and $n > m$, then there exists $x_0 \in S(\mathbb{R}^n)$ such that $g(x_0) = 0$. Bourgin [5] and Yang [24], [25] showed independently that in this situation the set

$$Z_g = \{x \in S(\mathbb{R}^n) \mid g(x) = 0\}$$

is of dimension at least $n - m - 1$.

The Borsuk–Ulam theorem proved to be one of the most useful tools of elementary algebraic topology. For this reason, it has numerous and far reaching extensions and generalizations, and continues to attract attention. Instead of repeating the story of why this is so, we refer the reader to a book by Matoušek [15] and/or a survey by Živaljević [26]. Let us only mention, very tersely, that Borsuk–Ulam theorems often allow to infer that combinatorial problems have a solution. From this perspective, Bourgin–Yang theorems yield information about the size of the set of those solutions. Interestingly enough, there are not nearly as many papers emphasising the latter point of view. Furthermore, to the best of our knowledge, those which exist rely on having a sphere as the (co)domain of the equivariant map in question (e.g. Marzantowicz,

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