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Couple stresses effect on instability and nonlinear stability in a double diffusive convection

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ABSTRACT

In this study, we have addressed the problem of double-diffusive convection in a reacting fluid with the effect of couple stresses. In this system, there are two competing effects which are the temperature gradation that leads to instability and a salt gradient which increases the stability of the system. The density is assumed to have quadratic dependence on the temperature and a linear dependence on the concentration. Linear instability and nonlinear stability analyses were performed. The standard energy method does not give an unconditional stability so a weighted energy analysis is used to achieve global results. Moreover, in addition to the weighted energy analysis, a global nonlinear stability analysis (unconditional) was proposed. The eigenvalue systems, which result from the linear and nonlinear theories, have been solved using the Chebyshev collocation method. Then, the accuracy of this method has been tested using the analytical solution of eigenvalue system for linear instability theory. The results show that the Chebyshev collocation method was very accurate even with high order derivatives which are produced by the couple stresses term. Finally, numerical results for the linear instability, weighted energy and global nonlinear thresholds were computed and discussed in detail.

1. Introduction

Convection problems which have two stratifying agents, such as heat and salt, with different diffusivities, existing in a fluid layer, have a multitude of important phenomena that cannot occur in a fluid layer which contains one such agency. In the past few decades, many theoretical and practical investigations have studied the convection in a fluid layer with two or more stratifying agencies. Turner [1–3], Huppert and Turner [4], and Platten and Legros [5] have provided excellent reviews of these studies. The marked difference between single component and multicomponent systems has given rise to an interest in the study of two or multi-component convection. In contrast to single component systems, convection begins even when the density decreases with height, i.e., when the basic state is hydrostatically stable. Double diffusive convection is of importance in a variety of fields such as high quality crystal production, liquid gas storage, oceanography, the production of pure medication, solidification of molten alloys, and geothermally heated lakes and magmas.

The association of heat and mass transfer with chemical reactions has many applications in practice, and therefore has attracted the attention of many researchers in recent years [6–8]. In processes such as drying, evaporation from the surface

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of a body of water, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in such industries as the power industry, where among electric power generation methods is one in which electrical energy is extracted directly from a moving conducting fluid.

Earlier in this work, the theoretical and experimental results on the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics, underwent a detailed review, conducted by Chandrasekhar [9]. Such investigations of these kinds of fluids are important, bearing in mind the increasing importance of non-Newtonian fluids in technology and industries. Stokes [10] has put forward the theory of couple-stress fluids. Couple-stresses are present in significant magnitude in fluids with very large molecules. Applications of couple-stress fluids occur in connection with the study of the mechanism of synovial joint lubrication, currently being focussed upon by researchers. A human joint is a dynamically loaded bearing with an articular cartilage as the bearing, and synovial fluid as the lubricant. The normal synovial fluid is clear or yellowish and is a non-Newtonian, viscous fluid. Because of the long chain of lauronic acid molecules found as additives in synovial fluid, Walicki and Walicka [11] modelled the fluid in question as couple-stress fluid in human joints. The issue of a couple-stress fluid and porous medium has also been investigated in [12–14]. Because of the numerical difficulties that are caused by couple stresses term, previous studies have adopted only the stress-free boundary conditions which assume that there is no mass flux across the boundary. This type of condition makes the finding of an analytical solution possible and therefore a numerical solution can be avoided. Moreover, these studies deal with problems which have eigenvalue systems which are free from variable coefficients, again to avoid numerical solution, which makes the analysis very poor and uninformative.

A double diffusive convection problem with no-slip boundary conditions and nonconstant coefficients in the system, which are believed to be highly relevant in real situations, is (we believe) analysed for the first time in this article. With the couple stresses effect, the eigenvalue system consists of three ordinary differential equations and one of these equations contains derivatives of sixth order. So we used transformations to make the system of the fourth order with five differential equations. We then applied the Chebyshev collocation method to approximate the system and then the accuracy of the numerical solution has been tested by using an analytical solution. Therefore, the problem which has been addressed in this study differs from other studies in that it uses no-slip boundary conditions, and the system contains nonconstant coefficients.

The objective of this article is to study both the linear and nonlinear stability theories of a double diffusive problem in a reacting fluid layer with the couple stresses effect. Finding stability and instability thresholds for convection problems and determining their type is important in understanding this system, which can be obtained by performing both the linear instability and the nonlinear stability theories of the model under study. Comparison of these thresholds allows us to assess the suitability of linear theory to predict convection thresholds. In order to find the results of stability, we draw our attention to the flexible energy method [15]. Nonlinear energy methods are useful because they delimit the parameter region of possible subcritical instability (the region between linear instability and nonlinear stability thresholds). Thus, measuring the difference between these two thresholds makes it possible to assess the suitability of linear theory to predict the stability of our convection problem. Recent contributions include the study of convective instabilities in fluid and porous media layers [16–32].

This paper is organized as follows. In Section 2, we will present mathematical formulas of the system with the associated perturbation equations. Then, we will introduce a linear instability analysis (Section 3), and a weighted and global nonlinear stability analysis (Section 4). Since stability analyses involve eigenvalue problems with non-constant coefficients, numerical methods are used for these problems. The appropriate numerical method will be explained in Section 5. The last section deals with the numerical results of linear theory and compares them directly with the weighted and global nonlinear theories.

2. Basic equations

We suppose the fluid is contained in the plane layer $\{z \in (0, d)\} \times \mathbb{R}^2$, and is incompressible, although a Boussinesq approximation is employed in the buoyancy term in the momentum equation. The z direction is denoted by the vector k with i, j, k being the standard Cartesian basis. Gravity acts in the negative z direction and we assume that the density ρ is constant, everywhere except the body force. Then, the Navier–Stokes equation for the fluid motion are

$$\rho_0(v_{i,t} + v_j v_{i,j}) = -p_{,i} + \beta \Delta v_i - \hat{\beta}\Delta^2 v_i - k_i g \rho_0 (1 - \alpha_t (T - T_m)^2 + \alpha_c (C - C_m)), \tag{2.1}$$

where ρ_0, $\mathbf{v}$, p, T, c are the reference value of density, velocity field, pressure, temperature field, and concentration of solute. Additionally, β is the dynamic viscosity, $\hat{\beta}$ is the couple stress viscosity, g is gravity, C_m is the reference value of concentration, and $\mathbf{k} = (0, 0, 1)$. T_m is that value of temperature where the density achieves a maximum. (Several real fluids display a density maximum, perhaps the most common being water, where $T_m \approx 4°C$). In this equation $\alpha_c = -\rho^{-1}\partial\rho/\partial C$ is a concentration expansion coefficient analogous to the thermal expansion coefficient $\alpha_t = -\rho^{-1}\partial\rho/\partial T$, where

$$\rho = \rho_0(1 - \alpha_t(T - T_m)^2 + \alpha_c(C - C_m)).$$

We assume that α_t and α_c are constants. Throughout, we use standard indicial notation and the Einstein summation convention so that e.g. $v_{i,t} = \partial v_i/\partial t$, and $p_{,i} = \partial p/\partial x_i$, $v_j v_{i,j} \equiv (\mathbf{v}.\nabla)\mathbf{v}$, and Δ is the Laplacian. The balance of mass equation is

$$v_{i,i} = 0. \tag{2.2}$$