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A new approximate method and its convergence for a strongly nonlinear problem governing electrohydrodynamic flow of a fluid in a circular cylindrical conduit

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ABSTRACT

In this paper, we propose a new approximate method, namely the discrete Adomian decomposition method (DADM), to approximate the solution of a strongly nonlinear singular boundary value problems describing the electrohydrodynamic flow of a fluid in an iron drag configuration in a circular cylindrical conduit. Convergence of the new method is analyzed. The velocity field of electrohydrodynamic flow of a fluid is determined. The influence of the Hartmann electric number and the strength of nonlinearity on the velocity profile is investigated. It is observed that the velocity field increases with the increase in the Hartmann electric number and decreases with the increase in the strength of nonlinearity. The results obtained by the proposed method are compared with results given in the literature.

1. Introduction

The aim of this paper is to present a novel approach to approximate the solution of a strongly nonlinear singular two-point boundary value problem governing the electrohydrodynamic flow of a fluid in an iron drag configuration in a circular cylindrical conduit (see Fig. 1) [1–3]:

$$(ru'(r))' = -rH^2\left(1 - \frac{u(r)}{1-\beta u(r)}\right), \tag{1}$$

subject to the boundary conditions:

$$u'(0) = 0, \quad u(1) = 0. \tag{2}$$

Here $u(r)$ is the fluid velocity, r is the radial distance from the center of the cylindrical conduit, H is the Hartmann electric number and the parameter β is a measure of the strength of the non-linearity and is related to the pressure gradient, the ion mobility and the current density at the inlet of the conduit. The Hartmann electric number H is defined by

$$H = (j_0 a^2/(\mu K^2 E_0))^{1/2}, \tag{3}$$

where j_0 is the uniform electrical current density at the inlet, a the radius of the conduit (before nondimensionalization), K the mobility of the ions, μ is the viscosity and E_0 the electric field. We note that the Hartmann number increases with decreasing viscosity μ.

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Nomenclature

u	fluid velocity
r	radial distance from the center of the cylindrical conduit
H	Hartmann electric number
β	strength of nonlinearity

Next, the nonlinearity parameter β is defined as

$$\beta = \frac{\partial p}{\partial z} K/j_0 - 1, \tag{4}$$

where $\frac{\partial p}{\partial z}$ is the pressure gradient, assumed constant. The electrohydrodynamic flow is useful in analysis of flow meters, accelerators and magneto-hydrodynamic generators. For a more detailed explanation of this problem, we refer the reader to [1]. In [2], Paullet discussed the existence and uniqueness results for problem (1) and (2).

It may be very difficult or impossible to find exact solution of problem (1) and (2), because the differential equation of problem (1) and (2) is highly nonlinear, has a singularity at the origin, and the type of nonlinearity in (1) is in the form of a rational function. Several different approaches have been proposed in the literature to handle the problem under consideration, both analytically and numerically. For instance, McKee et al. [1] developed perturbation solutions in terms of the parameter β and used a Gauss–Newton finite difference solver combined with the continuation method and a Runge–Kutta shooting method to provide numerical results for the fluid velocity over a large range of values of β. Mastroberardino [3] considered the application of homotopy analysis method (HAM) [4,5] to solve problem (1) and (2) for small values of β (especially for $\beta \in (0, 1]$). Ghasemi et al. [6] applied least square method to obtain an explicit analytic solution of electrohydrodynamic flow problem (1) and (2) for small and large values of β. In [7], Gavabari et al. presented Galerkin method, collocation approach and fourth-order Runge–Kutta method for the solutions of the problem under consideration for all values of relevant parameters. Pandey et al. [8] presented two semi-analytic techniques to approximate the solution of problem (1) and (2) based on optimal homotopy analysis method and optimal homotopy asymptotic method. Khan et al. [9] presented a new computational approach, namely, new homotopy perturbation method [10–14] to approximate the solution of problem (1) and (2). Mosayebidorcheh [15] applied the Taylor series solution based on differential transform method [16–19] to obtain an approximate solution of Eqs. (1) and (2) for $\beta \in (0, 1]$.

Adomian decomposition method [20] was firstly proposed by George Adomian and has been successfully employed in a variety of problem [21–27]. This method generates a solution in the form of a series with easily calculable components and converges rapidly in most cases. However, this method fails to provide convergent series solution for problem that contains strongly nonlinear term. Therefore, the classical ADM may be divergent when solving electrohydrodynamic flow problem (1) and (2). In this paper, we present a modified version of the ADM to solve problem (1) and (2). This modified method is named as the discrete Adomian decomposition method (DADM). This method is based on a combination of the domain decomposition approach and the classical ADM. The basic structure of the suggested method is very simple. There are three major steps occurring in the proposed DADM. In the first step, we discretize the domain of the problem into a finite number of subintervals of equal width. In the second step, we convert the given problem into an equivalent integral equation in each subinterval. Finally, we use classical ADM to solve the resulting integral equation subject to different left boundary conditions in each subinterval for obtaining the solution of problem (1) and (2).

Besides the construction of DADM, convergence analysis of the method is discussed. In addition, the velocity field of electrohydrodynamic flow of a fluid along the radial direction is computed. The influence of the Hartmann electric number and the strength of non-linearity on the velocity field is discussed. To illustrate the advantages of proposed method over

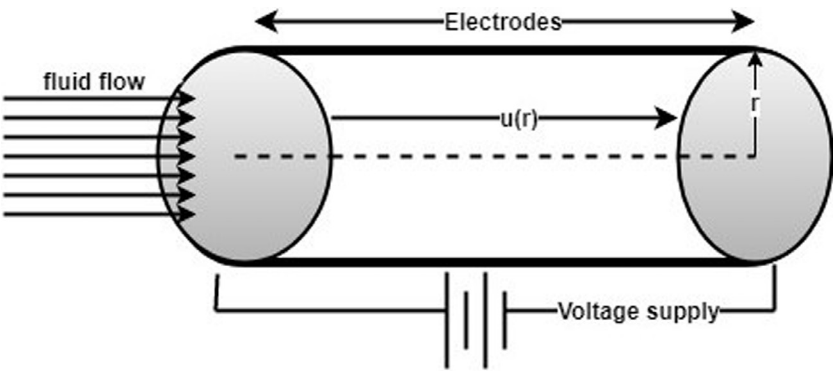

Fig. 1. Schematic diagram of electro-hydrodynamic flow in a circular cylindrical conduit.