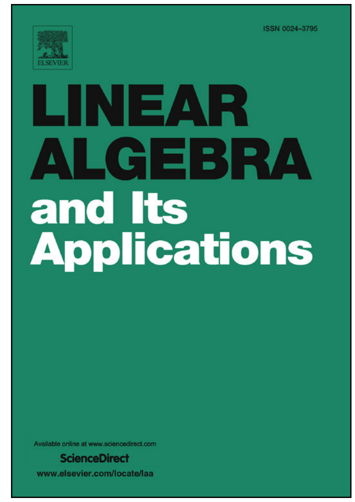
Accepted Manuscript

Eigenvalue location in graphs of small clique-width

Martin Fürer, Carlos Hoppen, David P. Jacobs, Vilmar Trevisan



PII: S0024-3795(18)30450-6
DOI: https://doi.org/10.1016/j.laa.2018.09.015
Reference: LAA 14730

To appear in: *Linear Algebra and its Applications*

Received date: 31 May 2018
Accepted date: 16 September 2018

Please cite this article in press as: M. Fürer et al., Eigenvalue location in graphs of small clique-width, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.09.015

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

EIGENVALUE LOCATION IN GRAPHS OF SMALL CLIQUE-WIDTH

MARTIN FÜRER, CARLOS HOPPEN, DAVID P. JACOBS, AND VILMAR TREVISAN

ABSTRACT. Finding a diagonal matrix congruent to $A-cI$ for constants c, where A is the adjacency matrix of a graph G allows us to quickly tell the number of eigenvalues in a given interval. If G has clique-width k and a corresponding k-expression is known, then diagonalization can be done in time $O(poly(k)n)$ where n is the order of G.

Keywords: adjacency matrix, eigenvalue, small clique-width.

1. INTRODUCTION

One of the main concerns of spectral graph theory is to determine properties of a graph through the eigenvalues of matrices associated with it. Even if we restrict ourselves to the adjacency and the Laplacian matrices, eigenvalues and eigenvectors have been particularly useful for isomorphism testing and embedding graphs in the plane [2], for graph partitioning and clustering [23], in the study of random walks on graphs [5, 22] and in the geometric description of data sets [6], just to mention a few examples. An obvious step in any such application is to calculate the spectrum of the input graph, or at least to accurately estimate a subset of its eigenvalues. In fact, the distribution of eigenvalues of graphs in a given class of graphs generated by a given random graph model has been studied intensively (see [11, 20, 24] and the references therein).

We say that an algorithm *locates eigenvalues for a class* $\mathcal{C}$ if, for any graph $G \in \mathcal{C}$ and any real interval I, it finds the number of eigenvalues of G in the interval I. In recent years, efficient algorithms have been developed for the location of eigenvalues in trees [15], threshold graphs [16] (also called *nested split graphs*), and chain graphs [1]. A rich class of graphs which contain threshold graphs are the graphs with no induced subgraph isomorphic to P_4, which are often called *P_4-free* graphs or *cographs*. Eigenvalue location in cographs and threshold graphs has been widely studied [29, 30, 31, 3, 17, 26].

It turns out that there is a strong connection between eigenvalue location and congruence of matrices, which we now describe. Two matrices R and S are *congruent*, which we write $R \cong S$, if there exists a nonsingular matrix P for which $R = P^T SP$. Let G be a graph with adjacency matrix A, and consider real numbers $c < d$. If we can construct a diagonal matrix $D_c \cong B = A - cI$, then Sylvester's Law of Inertia [25, p. 568] implies that the number n_1 of eigenvalues of A greater than c equals the number positive entries in D_c. (Similarly, the number of eigenvalues equal to c, or less than c, are given by the number of zero diagonal entries, or by the number of negative entries in D_c, respectively.) Hence, the number n_2 of positive entries in a diagonal matrix $D_d \cong A - dI$ is the number of eigenvalues of A greater than d. Thus $n_1 - n_2$ is the number of eigenvalues in $(c, d]$. This is why we want to design a fast algorithm to find a diagonal matrix that is congruent to $A - cI$.

1991 *Mathematics Subject Classification.* 05C50, 05C85, 15A18.
Key words and phrases. eigenvalues, clique-width, congruent matrices, efficient algorithms.

Download English Version:

https://daneshyari.com/en/article/11024727

Download Persian Version:

https://daneshyari.com/article/11024727

Daneshyari.com