Journal of Algebra 516 (2018) 298–328

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Universal enveloping associative Rota–Baxter algebras of preassociative and postassociative algebra

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A R T I C L E   I N F O

Article history:
Received 21 August 2017
Available online 24 September 2018
Communicated by Alberto Elduque

Keywords:
Rota–Baxter algebra
Universal enveloping algebra
PBW-pair of varieties
Preassociative algebra
Postassociative algebra

A B S T R A C T

Universal enveloping Rota–Baxter algebras of preassociative and postassociative algebras are constructed. The question of Li Guo is answered: the pair of varieties $(\mathrm{RB}_\lambda\mathrm{As}, \mathrm{postAs})$ is a PBW-pair and the pair $(\mathrm{RBAs}, \mathrm{preAs})$ is not.

Introduction

Linear operator R defined on an algebra A over the key field $\Bbbk$ is called Rota–Baxter operator (RB-operator, for short) of a weight $\lambda \in \Bbbk$ if it satisfies the relation

$$R(x)R(y) = R(R(x)y + xR(y) + \lambda xy), \quad x, y \in A. \tag{1}$$

An algebra with given RB-operator acting on it is called Rota–Baxter algebra (RB-algebra, for short).

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https://doi.org/10.1016/j.jalgebra.2018.09.017
0021-8693/

G. Baxter defined (commutative) RB-algebra in 1960 [4], solving an analytic problem. The relation (1) with $\lambda = 0$ appeared as a generalization of the integration by parts formula. J.-C. Rota and others [32,9] studied combinatorial properties of RB-operators and RB-algebras. In 1980s, the deep connection between Lie RB-algebras and Yang–Baxter equation was found [5,33]. To the moment, there are a lot of applications of RB-operators in mathematical physics, combinatorics, number theory, and operad theory [11,12,15,21].

There exist different constructions of free commutative RB-algebra, see the articles of J.-C. Rota, P. Cartier, and L. Guo [32,9,25]. In 2008, K. Ebrahimi-Fard and L. Guo obtained free associative RB-algebra [16,17]. In 2010, L.A. Bokut et al. [8] got a linear basis of free associative RB-algebra with the help of Gröbner–Shirshov technique. Diverse linear bases of free Lie RB-algebra were recently found in [20,23,31].

Pre-Lie algebras were introduced in 1960s independently by E.B. Vinberg, M. Gerstenhaber, and J.-L. Koszul [35,18,26], pre-Lie algebras satisfy the identity $(x_1x_2)x_3 - x_1(x_2x_3) = (x_2x_1)x_3 - x_2(x_1x_3)$.

J.-L. Loday [28] defined the notion of (associative) dendriform dialgebra, we will call it preassociative algebra or associative prealgebra. Preassociative algebra is a vector space with two bilinear operations $\succ, \prec$ satisfying the identities

$$(x_1 \succ x_2 + x_1 \prec x_2) \succ x_3 = x_1 \succ (x_2 \succ x_3), \quad (x_1 \succ x_2) \prec x_3 = x_1 \succ (x_2 \prec x_3),$$
$$x_1 \prec (x_2 \succ x_3 + x_2 \prec x_3) = (x_1 \prec x_2) \prec x_3.$$

In [27], J.-L. Loday also defined Zinbiel algebra (we will call it as precommutative algebra), on which the identity $(x_1 \succ x_2 + x_2 \succ x_1) \succ x_3 = x_1 \succ (x_2 \succ x_3)$ holds. Any preassociative algebra with the identity $x \succ y = y \prec x$ is a precommutative algebra and with respect to the new operation $x \cdot y = x \succ y - y \prec x$ is a pre-Lie algebra. In September 2018, it was proven in [13] that the pair of varieties of pre-Lie and preassociative algebras is a Poincaré–Birkhoff–Witt pair (PBW-pair) [30].

In [29], there was also defined (associative) dendriform trialgebra, i.e., an algebra with the operations $\prec, \succ, \cdot$ satisfying certain 7 axioms. (We will call such algebra as postassociative algebra or associative postalgebra.) Post-Lie algebra [34] is an algebra with two bilinear operations $[,]$ and $\cdot$; moreover, Lie identities with respect to $[,]$ hold and the next identities are satisfied:

$$(x \cdot y) \cdot z - x \cdot (y \cdot z) - (y \cdot x) \cdot z + y \cdot (x \cdot z) = [y, x] \cdot z, \quad x \cdot [y, z] = [x \cdot y, z] + [y, x \cdot z].$$

Given a binary operad $\mathcal{P}$, the notion of successor [2] provides the defining identities for pre- and post-$\mathcal{P}$-algebras. Equivalently, one can define the operad of pre- and post-$\mathcal{P}$-algebras as $\mathcal{P} \bullet \mathrm{PreLie}$ and $\mathcal{P} \bullet \mathrm{PostLie}$ respectively. Here PreLie denotes the operad of pre-Lie algebras and PostLie — the operad of post-Lie algebras, $\mathcal{V} \bullet \mathcal{W}$ is the black Manin product of operads $\mathcal{V}$, $\mathcal{W}$ (see [19] about operads and Manin products).