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CONSTRAINT METRIC APPROXIMATIONS AND EQUATIONS IN GROUPS

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Abstract. We introduce notions of a constraint metric approximation and of a constraint stability of a metric approximation. This is done in the language of group equations with coefficients. We give an example of a group which is not constraintly sofic. In building it, we find a sofic representation of free group with trivial commutant among extreme points of the convex structure on the space of sofic representations.

We consider the centralizer equation in permutations as an instance of this new general setting. We characterize permutations $p \in S_k$ whose centralizer is stable in permutations with respect to the normalized Hamming distance, that is, a permutation which almost centralizes p is near a centralizing permutation. This answers a question of Gorenstein-Sandler-Mills (1962).

1. Introduction

The concept of an equation is fundamental in mathematics. Usual attributes of an equation are variables and coefficients, and one searches for a solution subject to some admissibility condition or constraint. In group theory, the literature on equations with solutions in (and over) groups is immense.

We deal with *almost* solutions of equations in groups, where 'almost' is expressed in terms of a chosen bi-invariant metric on a group. This is very much relevant to the currently fast-developing area of metric approximations of groups. Notable examples of metrically approximable groups are sofic and hyperlinear groups. Sofic groups are groups approximable by $(S_k, d_H)_{k\in\mathbb{N}}$, symmetric groups of finite degree endowed with the normalized Hamming distance d_H. They have been introduced by Gromov in the context of Gottschalk's surjunctivity conjecture in symbolic dynamics and have been named 'sofic' by Weiss. Hyperlinear groups are groups approximable by $(U(k), d_{HS})_{k\in\mathbb{N}}$, unitary groups of finite rank endowed with the normalized Hilbert-Schmidt distance d_{HS}. They have been appeared in relation to Connes' embedding conjecture in operator algebras and have been named 'hyperlinear' by Rădulescu.

The *existence* of a metric approximation of a group G can be formulated through the existence of almost solutions of equations (without coefficients), defined in the approximating groups by the relator words of G. On the other hand, the *stability*

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