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Review

Anisotropic solitary semivortices in dipolar spinor condensates controlled by the two-dimensional anisotropic spin-orbit coupling

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ABSTRACT

We study families of anisotropic solitary semivortices (SVs) in spinor dipolar Bose-Einstein condensates (BECs), with two localized components linearly mixed by the two-dimensional anisotropic spin-orbit (SO) coupling of the Rashba type. The intrinsic nonlinearity of this system represented by the anisotropic dipole-dipole interactions (DDIs) between atomic magnetic moments is aligned parallel to the system's plane by an external magnetic field. The affects of the anisotropic SO-coupling and the interplay between SO-coupling and DDI to the solitons of the SV type are systematically studied through the paper. For the SVs, we demonstrate that the shape of the vortex component, which is vertical or horizontal anisotropic solitary vortices (ASV) with respect to the in-plane polarization of the atomic dipole moments in the underlying BEC, may be effectively controlled by the anisotropy degree, λ_x/λ_y, of the SO-coupling. A novel type of ASV, elliptical ring shaped vortices, is found at the transition point between the horizontal and vertical shapes. Such ASV contains the largest value of the average angular momentum. Influences of the anisotropic SO-coupling to the mobility of the SV are studied by the direct simulation to the kicked soliton on the x and y directions. Trajectories of the kicked soliton show a strong anisotropy which depends on the anisotropy degree of the SO-coupling.

1. Introduction

Spin-orbit (SO) coupling is a well-known effect, which has already been extensively studied in many branches of physics. It can give rise to fine structure splitting, which plays an important role in atomic physics. It is also one of the major effects to control the electron transport in a semiconductor in condensed matter physics. Recently it was found that SO-coupling can cause novel physics such as anomalous quantum Hall effects [1], topological insulators [2], and topological superconductors [3], etc. In recent years, the study of the effect induced by the SO-coupling on quantum gas has become one of the hottest topics since the synthetic SO-coupling was generated in ultra cold atoms. Lots of theoretical and experimental progress have been made in this direction, which are summarized in Reviews [4,5]. Recent experiments have successfully realized SO-coupling in two-dimensional (2D) space [6–8]. Because the cold atom systems can reach a strongly interacting quantum gas by using the Feshbach resonance [9,10], the interplay between the SO coupling and the collisional nonlinearity of the BEC gives rise to diverse nonlinear phenomena such as one-dimensional (1D) solitons [11–20], two-dimension (2D) gap solitons [21], stripe phases [22,23], etc. In three-dimensional (3D) settings, SO-coupling also plays an important role in the formation of complex topological modes in BEC [24,25], such as skyrmions [26–28].

Recently, an unexpected result was revealed by the analysis of a two-component (spinor) self-attractive BEC with linear SO-coupling between the components which predicts two types of completely stable 2D solitons, namely, semi-vortices (SVs) and mixed modes (MMs) [29–37]. The semi-vortices are built of one zero-vorticity and one vortical components, while the MMs mix zero and non zero vorticities in both components (which are built as equal-weight superpositions of SVs with topological content $(0,-1)$ and $(0,+1)$ in the two components). These findings contradict the common belief that any system with pure cubic self-focusing nonlinearity cannot support stable solitons in free 2D space. In 3D scheme, it was traditionally believed that no metastable localized mode can exist in free 3D space with cubic attractive nonlinearity, however, it has been demonstrated that the

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interplay between the linear Wely-type SO-coupling and the cubic attractive interaction gives rise to the same two generic types of 3D solitons as in 2D, i.e., SVs and MMs, which are metastable states in free space [38]. The SO-coupling like effect was also considered in some special optical system to create the spatiotemporal solitons [39–41].

The interplay between SO-coupling and nonlocal nonlinearity was also studied by means of dipolar BECs. It was reported that stripe soliton and anisotropic vortex can be created by the combination action of the anisotropic dipole-dipole interactions (DDIs) and SO-coupling [42,43], and 2D gap solitons can be created with Zeeman splitting [44,45]. Recently, it has been demonstrated that the pattern and the mass of the anisotropic vortex soliton can also be controlled by the Zeeman splitting [47] and the mixture of Rashba and Dresselhaur type SO couplings [46]. In above 2D models, the SO-coupling strength in the x and y directions are equal. For example, a typical Rashba-type SO coupling can be described by $(\lambda_x\hat{k}_x\hat{\sigma}_y + \lambda_y\hat{k}_y\hat{\sigma}_x)$, where the coupling strengths are always be taken as $\lambda_x = \lambda_y$. However, recent experiment has demonstrated that the strength $\lambda_{x,\,y}$ can be tuned to be unequal with each other [7], which results in an anisotropic 2D SO-coupling. How such an anisotropic 2D SO-coupling affects the soliton has not yet been considered.

In this work, we consider the dynamics of the dipolar matter-wave soliton in 2D BECs with anisotropic Rashba SO-coupling (i.e., $\lambda_x \neq \lambda_y$) to study how the anisotropic SO-coupling affects the SVs. Because the dipolar BECs can feature an anisotropic long-range interaction by means of the DDIs, the interplay between these two kinds of anisotropy, DDIs and SO-coupling, is also considered in this work. The rest of the paper is structured as follows. The model is introduced in Section 2. Basic numerical results for the formation and dynamics of SVs against the anisotropic SO-coupling are reported in Section 3 and 4. The paper is concluded by Section V.

2. The model

Under the mean-field approximation, the evolution of the wave functions of the spinor dipolar BECs, $\psi = (\psi_+, \psi_-)$, is governed by the coupled Gross–Pitaevskii equations:

$$\begin{aligned} i\partial_t\psi_+ &= -\frac{1}{2}\nabla^2\psi_+ + \hat{D}^{[-]}\psi_- + \psi_+\int dx'dy'R(x-x',y-y') \\ &\quad (|\psi_+(x',y')|^2 + |\psi_-(x',y')|^2), \\ i\partial_t\psi_- &= -\frac{1}{2}\nabla^2\psi_- - \hat{D}^{[+]}\psi_+ + \psi_-\int dx'dy'R(x-x',y-y') \\ &\quad (|\psi_+(x',y')|^2 + |\psi_-(x',y')|^2). \end{aligned} \tag{1}$$

which is written in the normalized form with the anisotropic Rashba SO-coupling term

$$\hat{D}^{[\pm]} = \lambda_x\partial_x \pm i\lambda_y\partial_y, \tag{2}$$

where $\lambda_{x,\,y} \in [0,\,1]$ are the strength of SO-coupling in x and y directions, respectively. Here, we fix the maxima of the strengths to be 1 by means of rescaling. The DDI's kernel reads

$$R(x-x',y-y') = \frac{1-3\cos^2\theta}{[\epsilon^2 + (x-x')^2 + (y-y')^2]^{3/2}}, \tag{3}$$

where cutoff ϵ is determined by the confinement in the transverse (third) dimension [51,52]. So if the dipoles are polarized (by an external magnetic field) in the positive x direction in the 2D $(x,\,y)$ plane, $\cos^2\theta = (x-x')^2/\left[(x-x')^2 + (y-y')^2\right]$, and if the dipoles are polarized in the positive y direction, $\cos^2\theta = (y-y')^2/\left[(x-x')^2 + (y-y')^2\right]$.

In order to study the interplay between two types of anisotropy (i.e. anisotropic DDIs and SOC) clearer, we neglect the contact interaction from Eq. (1), which is assumed to be achieved by tuning Feshbarch resonance [9,10]. Because the collapse of soliton can be arrested by the DDIs [48,49], collapse of soliton in usual SO-coupling system with contact interaction when total norm of the soliton exceeds the limit of Townes soliton [29,50] is never observed in this system.

Stationary states are look for as the usual form, $\psi_\pm(x,y,t) = \phi_\pm(x,y)e^{-i\mu t}$, where $\phi_\pm$ are stationary wave functions, and μ is a real chemical potential. A dynamical invariant of the system is the total norm, which is proportional to the total number of atoms in the binary BECs:

$$N = N_+ + N_- = \int dxdy(|\phi_+|^2 + |\phi_-|^2). \tag{4}$$

It is also relevant to define the relative share of the atoms which are kept in the vortex component:

$$F_- = N_-/(N_+ + N_-). \tag{5}$$

Here, we select $\epsilon = 0.5$, which is the same as what had been selected in Refs. [21,35,47]. According to the physical estimation in Refs. [21,35], $\{x,\ y\}$=1–10 μm, and N in the range of 1 to 5 corresponds to the number of the atoms 10^4–10^5.

The other dynamical invariants are the linear momentum [see Eq. (13) below] and the total energy,

$$E = E_{\mathrm{K}} + E_{\mathrm{DDI}} + E_{\mathrm{SOC}}, \tag{6}$$

where E_{K}, E_{DDI}, and E_{SOC} are the kinetic energy and the interaction energy through DDI and SO-coupling, respectively:

$$\begin{aligned} E_{\mathrm{K}} &= \frac{1}{2}\int dxdy\left(|\nabla\phi_+|^2 + |\nabla\phi_-|^2\right), \\ E_{\mathrm{DDI}} &= \frac{1}{2}\iint dxdydx'dy'\left(|\phi_+(x,y)|^2 + |\phi_-(x,y)|^2\right)R(x-x',y-y') \\ &\quad (|\phi_+(x',y')|^2 + |\phi_-(x',y')|^2), \\ E_{\mathrm{SOC}} &= \int dxdy(\phi_+^*\hat{D}^{[-]}\phi_- - \phi_-^*\hat{D}^{[+]}\phi_+). \end{aligned} \tag{7}$$

Note that although the anisotropic system does not conserve the total orbital angular momentum, the expecting value of the orbital angular momentum operator acts on each component,

$$\langle L_\pm \rangle = \frac{1}{N_\pm}\int \phi_\pm^*\hat{L}\phi_\pm dxdy, \tag{8}$$

where $\hat{L} = -i(y\partial_x - x\partial_y)$ is the angular-momentum operator, can still be used to characterize the degree of vorticity of the components.

Bright-soliton modes of the semi-vortex type in various systems may be produced by guess input [29],

$$\phi_+^0 = A_+\exp(-\alpha_+r^2), \quad \phi_-^0 = A_-r\exp(i\theta - \alpha_-r^2), \tag{9}$$

where $A_\pm$ and $\alpha_\pm$ are positive constants. In this guess, ϕ_+ and ϕ_- are the wave functions of zero-vorticity (fundamental) and vortex components, respectively. Starting from this input, SVs can be produced by the imaginary-time-integration method [53–55]. Because the dipole-dipole interaction can stabilize soliton in 2D, the soliton we obtained in this paper are all stable.

In the following sections, we numerically identify different types of soliton by varying 3 control parameters, *viz*., the total norm of the soliton, as given by Eq. (4), and the SO-coupling strength in the two directions, i.e., $\lambda_{x,\,y}$ in Eq. (1).

3. Semi-vortex solitons

We firstly consider all the dipoles are directed to the positive x-axis. Figs. 1 and 2 display typical examples of SVs by varying λ_x and λ_y, respectively. The first row of Figs. 1 and 2 show that the vortical component ϕ_- forms a vertical and horizontal dipole modes at $(\lambda_x, \lambda_y) = (0, 1)$ and (1,0), respectively, that is the SO-coupling retreats to a single direction (i.e., 1D version) in y and x,