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A fractional model of vertical transmission and cure of vector-borne diseases pertaining to the Atangana–Baleanu fractional derivatives

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ABSTRACT

The model of transmission dynamics of vector-borne diseases with vertical transmission and cure within a targeted population is extended to the concept of fractional differentiation and integration with non-local and non-singular fading memory introduced. The effect of vertical transmission and cure rate on the basic reproduction number is shown. The Atangana–Baleanu fractional operator in caputo sense (ABC) with non-singular and non-local kernels is used to study the model. The existence and uniqueness of solutions are investigated using the Picard–Lindel method. Ultimately, for illustrating the acquired results, we perform some numerical simulations and show graphically to observe the impact of the arbitrary order derivative. It is expected that the proposed model will show better approximation than the classical model established before.

1. Introduction

Vector-borne diseases are infectious diseases which are transmitted to humans and animals by blood-feeding arthropods. Some well known vector-borne diseases include West dengue fever, Nile virus, malaria, viral encephalitis e.t.c [1]. The diseases are caused by pathogens like parasites, bacteria and viruses. Arthropods insects that suck the human and animals blood. Such insects includes mosquitoes, biting flies, ticks, e.t.c [2]. They transmit the pathogens to human host by carrying them from an infected host.Vector-borne diseases are more common places with hot weather conditions, like tropics and sub-sahara deserts. The diseases are some of the most relevant cause of global health illnesses and a lot of them are killer diseases [3]. The World Health Organization (WHO)reported the numbers of deaths in different parts of the world yearly. Almost 700 million humans get infected by mosquito-borne sicknesses out of which about one million deaths are recorded annually [4,5]. Based on the above facts, one can see that it is vital to control these diseases. Therefore, it is important to understand the dynamic behaviors of the diseases in order to come up with a comprehensive treatment of the infected hosts [5].

Fractional operators have been applied to model a lot of problems [8]. They have been applied in engineering, science, etc. A lot of models have been treated using different fractional operators [7–15]. It is well known that Caputo and Riemann Liouville have singular kernels. To tackle the problem of singularity, a new fractional derivative was proposed in [9]. The Atangana–Baleanu derivative possesses the futures of the Caputo and Fabrizio with the kernel being non-local and non singular [9]. The aim of the application of the Atangana–Baleanu fractional operator is to include into the mathematical formulation of the dynamical system the effect of non-local fading memory. Recent developments pertaining to this important derivative have been reported in [11–13]. One of the application of this operator is modelling biological problems is that, the physical problem within a targeted population follow the Mittag–Leffler law as the it is not non-local. Unlike the power law kernel used in the classical Riemann–Liouville and Caputo derivative, the Mittag–Leffler guarantees no singularity; this helps us to have a clear knowledge of the beginning and of the end of the evolution of the model under consideration. Therefore, Eq. (1) will be extended using this differential operator with the Mittag–Leffler kernel. Recent studies in numerical and analytical techniques of

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Table 1
Description of model parameters in Eq. (1).

$S(t)$	Size of susceptible population.
$I(t)$	Size of infectious population.
$T(t)$	Size of population under treatment.
$R(t)$	Size of recovered population.
$V(t)$	Size of susceptible vector population.
$W(t)$	Size of infected vector population.
ϵ_1	Fraction of human host given birth by infected parents in b_1.
β_1	Rate of direct transmission of the disease.
β_2	The vector mediated transmission rate.
α	Rate at which infectious humans are treated.
ϵ_2	Fraction of human host given birth by infected parents in b_2.
μ_1	is the natural mortality rate of a human.
μ_2	Rate of natural mortality of the vector population.
δ_1	Rate of suffering of the disease leading to death in some instances.
δ_2	Rate at which Infectious vectors die.
ηI	Natural recovery rate.
b_1	Constant birth rate at which human host population are recruited.
b_2	Constant recruitment rate of vector population.
γ	Rate of recovery of treated humans.
β_3	Rate at which mosquitoes become infected upon biting infected human.

non-local kernel are getting interesting to researchers studying in this field.The classical form of the model was recently proposed in [5]. The global stability analysis and numerical simulation of the model were studied. The main suppose of this study is to expand the model under consideration by replacing the derivative with the ABC [5].

In this work, we aim to study the uniqueness and existence of special solutions of the model under the Atangana–Baleanu operator using the Picard–Lindelof method [8]. Then, numerical simulation of the model will be put in place using a newly established numerical scheme for the Atangana–Baleanu fractional operator [10]. The model is represented by the nonlinear system of ordinary differential equations given by [5]:

$$\begin{cases} \frac{dS(t)}{dt} = (1-\epsilon_1 I)b_1 - \beta_1 SY - \beta_2 SW - \mu_1 S, \\ \frac{dI(t)}{dt} = \epsilon_1 b_1 I + \beta_1 SI + \beta_2 SW - \alpha I - \eta I - \delta_1 I - \mu_1 I, \\ \frac{dT(t)}{dt} = \alpha I - \gamma I - \delta_1 T - \mu_1 T, \\ \frac{dR(t)}{dt} = \eta I + \gamma T - \mu_1 R, \\ \frac{dV(t)}{dt} = (1-\epsilon_2 W)b_2 + \beta_3 VI - \mu_2 W, \\ \frac{dW(t)}{dt} = \epsilon_2 b_2 W + \beta_3 VI - \delta_2 W - \mu_2 W. \end{cases} \tag{1}$$

with the initial conditions $S(0) \geq 0$, $I(t) \geq 0$, $T(0) \geq 0$, $R(0) \geq 0$, $V(0) \geq 0$, $W(0) \geq 0$, where the parameters in the model are defined in the Table 1 [6]. Fig. 1 shows the flow chart represents the interactions and transfer of a vector-borne disease in both human and vector populations.

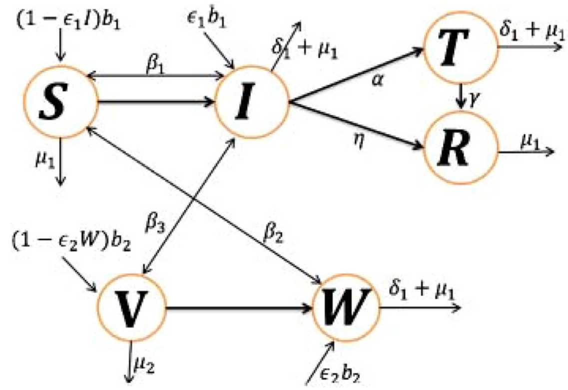

Fig. 1. The flow chart of the model parameters interactions [6].

2. Fractional order derivative with Mittag–Leffler kernel

In this part, we consider some properties and definitions of the new fractional derivatives [5].

Definition 1. Let $f \in K'(b, d)$, $d > b$, $\rho \in [0, 1]$, the AB fractional operator in Caputo sense (ABC) can be represented by:

$$ {}^{ABC}_{b}D^{\rho}_{t} f(t) = \frac{F(\rho)}{1-\rho} \int_b^t f'(\tau) E_\rho \left[-\rho \frac{(t-\tau)^\rho}{1-\rho} \right] d\tau. \tag{2}$$

In Eq. (2), $F(\rho)$ is a normalized function with $F(0) = F(1) = 1$.

Definition 2. A fractional integral with order ρ of a fractional operator is represented by the following

$$ {}^{AB}_{b}D^{\rho}_{t} f(t) = \frac{F(\rho)}{1-\rho} \frac{d}{dt} \int_b^t f(\tau) E_\rho \left[-\rho \frac{(t-\tau)^\rho}{1-\rho} \right] d\tau. \tag{3}$$

Definition 3. Consider $f \in K'(b, d)$, $d > b$, $\rho \in [0, 1]$, which may not be differentiable, the AB fractional operator in Riemann–Liouville (ABR) sense can be represented by:

$$ {}^{ABR}_{b}D^{\rho}_{t} f(t) = \frac{1-\rho}{F(\rho)} f(t) + \frac{\rho}{F(\rho)\Gamma(\rho)} \int_b^t f(\tau)(t-\tau)^{\rho-1} d\tau. \tag{4}$$

2.1. Properties of the Atangana–Baleanu derivative

The above definitions have been applied to model a lot of real-world application. Consider the relationship between the derivatives and Laplace transform as given in [9]:

$$ \mathcal{L}\left\{ {}^{ABR}_{0}D^{\rho}_{t} f(t) \right\}(p) = \frac{F(\rho)}{1-\rho} \frac{p^\rho \mathcal{L}\{f(t)\}(p)}{p^\rho + \frac{\rho}{1-\rho}}. \tag{5}$$

and

$$ \mathcal{L}\left\{ {}^{ABC}_{0}D^{\rho}_{t} f(t) \right\}(p) = \frac{F(\rho)}{1-\rho} \frac{p^\rho \mathcal{L}\{f(t)\}(p) - P^{\rho-1} f(0)}{p^\rho + \frac{\rho}{1-\rho}}. \tag{6}$$

Theorem 2.1. *Let f be continuous on* $[b, d]$. *The following relation holds on* $[b, d]$ [7]

$$ \left\| {}^{ABR}_{b}D^{\rho}_{t} f(t) \right\| < \frac{B(\rho)}{1-\rho} \| f(x) \|, \tag{7}$$

here, $\| f(x) \| = max_{b \leq x \leq d} |f(x)|$.

Theorem 2.2. *The following Lipschitz conditions are satisfied for both (ABC)and (ABR) for any two functions* $f(t)$ *and* $g(t)$ [7]:

$$ \left\| {}^{ABR}_{0}D^{\rho}_{t} f(t) - {}^{ABR}_{0}D^{\rho}_{t} g(t) \right\| \leq H \| f(t) - g(t) \|, \tag{8}$$

and

$$ \left\| {}^{ABC}_{0}D^{\rho}_{t} f(t) - {}^{ABC}_{0}D^{\rho}_{t} g(t) \right\| \leq H \| f(t) - g(t) \|. \tag{9}$$

Theorem 2.3. *The following fractional ODE*

$$ {}^{ABC}_{0}D^{\rho}_{t} f(t) = u(t) \tag{10}$$

has the following unique solution [5]:

$$ f(t) = \frac{1-\rho}{F(\rho)} u(t) + \frac{\rho}{F(\rho)\Gamma(\rho)} \int_b^t u(\tau)(t-\tau)^{\rho-1} d\tau. \tag{11}$$