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$SU(3)$ quantum spin ladders as a regularization of the $\mathbb{C}P(2)$ model at non-zero density: From classical to quantum simulation

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ABSTRACT

Quantum simulations would be highly desirable in order to investigate the finite density physics of QCD. $(1+1)$-d $\mathbb{C}P(N-1)$ quantum field theories are toy models that share many important features of QCD: they are asymptotically free, have a non-perturbatively generated massgap, as well as θ-vacua. $SU(N)$ quantum spin ladders provide an unconventional regularization of $\mathbb{C}P(N-1)$ models that is well-suited for quantum simulation with ultracold alkaline-earth atoms in an optical lattice. In order to validate future quantum simulation experiments of $\mathbb{C}P(2)$ models at finite density, here we use quantum Monte Carlo simulations on classical computers to investigate $SU(3)$ quantum spin ladders at non-zero chemical potential. This reveals a rich phase structure, with single- or double-species Bose–Einstein "condensates", with or without ferromagnetic order.

1. Introduction

Monte Carlo simulations of Wilson's lattice QCD [1] are very successful in addressing static properties of hadrons [2,3] as well as the equilibrium thermodynamics of quarks and gluons at zero baryon density [4,5]. The real-time dynamics and the non-zero density physics of QCD [6], on the other hand, remain largely unexplored, because Monte Carlo simulations then suffer from very severe sign and complex action problems. Quantum simulation experiments are very promising for addressing these challenging questions, because quantum hardware (whose dynamics naturally incorporates quantum entanglement) does not suffer from such problems [7–12]. Indeed, quantum

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simulation experiments have already been carried out successfully in the context of condensed matter physics. In particular, the real-time evolution through a quantum phase transition in the bosonic Hubbard model, which separates a Mott insulator from a superfluid, has been realized in quantum simulation experiments with ultracold bosonic atoms in an optical lattice [13]. Similar experiments with fermionic atoms aim at quantum simulations of the fermionic Hubbard model, in the context of high-temperature superconductivity. The current experiments with fermionic gases have not yet succeeded to reach sufficiently low temperatures to explore the possible existence of high-temperature superconductivity in the fermionic Hubbard model. However, medium-range antiferromagnetic correlations have already been observed [14].

These impressive developments in the quantum simulation of condensed matter systems provide a strong motivation to explore the feasibility of quantum simulation experiments of QCD and other quantum field theories relevant in particle physics. While it seems difficult to embody Wilson's lattice QCD in ultracold quantum matter, an attractive alternative lattice regularization of QCD and other asymptotically free field theories is provided by quantum link models [15–19]. Quantum links are generalized quantum spins (associated with the links of a lattice) with an exact gauge symmetry. Wilson's link variables are classical $SU(3)$-valued parallel transporter matrices with an infinite-dimensional link Hilbert space. $SU(3)$ quantum links are again 3×3 matrices, but their matrix elements are non-commuting operators that act in a finite-dimensional link Hilbert space. This makes quantum link models ideally suited for quantum simulation experiments in which a finite number of quantum states of ultracold matter can be controlled successfully [20]. Indeed, quantum simulation experiments of Abelian [21–24] and non-Abelian gauge theories [25–27], some based on quantum link models, have already been proposed. In particular, ultracold alkaline-earth atoms in an optical superlattice [27] are natural physical objects that can embody non-Abelian $U(N)$ and $SU(N)$ gauge theories.

While first quantum simulation experiments of relatively simple Abelian and non-Abelian lattice gauge theories are expected in the near future, the quantum simulation of QCD remains a long-term goal [28]. The quantum link regularization of QCD [18] involves an additional spatial dimension (of short physical extent) in which the discrete quantum link variables form emergent continuous gluon fields via dimensional reduction. The extra dimension also gives rise to naturally light domain wall quarks with an emergent chiral symmetry. Incorporating these important dynamical features in quantum simulation experiments will be challenging, but does not seem impossible. In particular, synthetic extra dimensions have already been realized in quantum simulation experiments with alkaline-earth atoms [29].

In order to explore the feasibility of quantum simulation experiments of QCD-like theories, it is natural to investigate $(1+1)$-d $\mathbb{C}P(N-1)$ models [30,31]. These quantum field theories share crucial features with QCD: they are asymptotically free, have a non-perturbatively generated massgap, as well as non-trivial topology and hence θ-vacuum states. In particular, the $\mathbb{C}P(N-1)$ model has a global $SU(N)$ symmetry that gives rise to interesting physics at non-zero density, which can be explored via chemical potentials. As in QCD, the direct classical simulation of $\mathbb{C}P(N-1)$ model θ-vacua, finite density physics, or dynamics in real-time suffer from severe sign problems, and thus strongly motivate the need for quantum simulation.[1] Again, alkaline-earth atoms in an optical superlattice are natural degrees of freedom to realize the $SU(N)$ symmetry of $\mathbb{C}P(N-1)$ models [35,36].

In complete analogy to the quantum link regularization of QCD, $(1+1)$-d $\mathbb{C}P(N-1)$ models can be regularized using $(2+1)$-d $SU(N)$ quantum spin ladders [37,38]. Again, there is an extra spatial dimension of short physical extent in which the discrete quantum spins form emergent continuous $\mathbb{C}P(N-1)$ fields via dimensional reduction. The continuum limit of the $(1+1)$-d $\mathbb{C}P(N-1)$ quantum field theory is taken by gradually increasing the extent L' of the extra dimension. Thanks to asymptotic freedom, this leads to an exponential increase of the correlation length $\xi \gg L'$ in the physical dimension, and thus to dimensional reduction from $(2+1)$-d to $(1+1)$-d, similar to the $O(3) = \mathbb{C}P(1)$ model [39,40]. All this is analogous to QCD, but has the great advantage that it can already be investigated with currently available experimental quantum simulation techniques. In particular, by

[1] It should be noted that classical sign-problem-free simulations of $\mathbb{C}P(N-1)$ models at non-zero density are possible after an analytic rewriting of the partition function [32–34]. Unfortunately, this does not seem to extend to QCD.