Physics Letters B 786 (2018) 249–254

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Radiative instability of quantum electrodynamics in chiral matter

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ARTICLE INFO

Article history:
Received 2 July 2018
Received in revised form 26 September 2018
Accepted 30 September 2018
Available online 2 October 2018
Editor: J.-P. Blaizot

ABSTRACT

Modification of the photon dispersion relation in chiral matter enables $1 \to 2$ scattering. As a result, the single fermion and photon states are unstable to photon radiation and pair production respectively. In particular, a fast fermion moving through chiral matter can spontaneously radiate a photon, while a photon can spontaneously radiate a fast fermion and anti-fermion pair. The corresponding spectra are derived in the ultra-relativistic approximation. It is shown that the polarization of the produced and decayed photons is determined by the sign of the chiral conductivity. Impact of a flat thin domain wall on the spectra is computed.

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1. Introduction

One of the macroscopic manifestations of the chiral anomaly of QCD is the emergence of the topological CP-odd domains in hot nuclear matter [1]. QED is coupled to these domains via its own chiral anomaly. This is represented by the triangular diagrams that involve two photon fields and the axial current generated by the topological fluctuations of the gluon field. The axial current rapidly increases with temperature which triggers a variety of non-trivial electromagnetic effects in quark–gluon plasma [2].

At a more fundamental level, the chiral anomaly makes photon effectively massive. Consequently, single photon and fermion states become unstable. Recall that photon radiation by a charged fermion in vacuum $f(p) \to f(p') + \gamma(k)$ and the cross-channel process of pair production in vacuum $\gamma(k) \to f(p') + \bar{f}(p)$ are prohibited by momentum conservation.[1] Indeed in the rest frame of one of the fermions $k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$. The right-hand-side never vanishes since $\varepsilon > m$, whereas in the left-hand-side $k^2 = 0$ [3].[2] In chiral matter, i.e. in a matter supporting the CP-odd domains, the chiral anomaly modifies the photon dispersion relation as [4–10][3]

$$k^2 = -\lambda \sigma_\chi |\boldsymbol{k}| \,, \tag{1}$$

where λ and $\boldsymbol{k}$ are photon helicity and momentum and σ_χ is the chiral conductivity [11–13]. This opens the $1 \to 2$ scattering channels, viz. the pair-production if $k^2 > 0$ and the photon radiation if $k^2 < 0$. Thus, single-particle states in chiral matter are unstable with respect to spontaneous radiation and decay. Moreover, in a matter with positive σ_χ, only the right-polarized photons with $\lambda = +1$ can be radiated, while only the left-polarized photons with $\lambda = -1$ decay and vice-versa in a matter with negative σ_χ.

The main goal of this paper is to compute the photon radiation and pair production spectra due to the modified photon dispersion relation in chiral matter. The fermions (and antifermions) are considered to be external particles propagating through the chiral matter. Their energy is assumed to be much larger than the medium ionization energy. The calculation method is borrowed from [15] and relies on several approximations: (i) photons and fermions are ultra-relativistic in the laboratory frame (the one associated with the matter), this means that $\varepsilon \gg m$, $\omega \gg \sigma_\chi$. Apart from making calculations significantly less bulky, this allows one to neglect the effect of the electromagnetic field instability in the infrared region as explained in Sec. 2. (ii) The matter is assumed to be spatially homogeneous and consisting of either one infinite CP-odd domain or of two semi-infinite domains separated by a flat domain wall.

The paper is structured as follows. In Sec. 2 and Sec. 3 the wave functions of an ultra-relativistic photon and fermion in chiral matter are derived. In Sec. 4 they are employed to compute the photon spectrum radiated by a fermion in chiral matter with constant σ_χ as well as in matter with two chiral domains separated by a thin flat domain wall. The cross-channel process of pair production is analyzed in Sec. 5. The results are summarized and discussed in Sec. 6.

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[1] p, p' and k are four-momenta with the components $p = (\varepsilon, \boldsymbol{p})$, $p' = (\varepsilon', \boldsymbol{p}')$ and $k = (\omega, \boldsymbol{k})$.

[2] In the limit of massless fermions, no production is possible as well, since in the only kinematically allowed configuration all particle momenta are strictly parallel.

[3] In covariant form $k^2 = -\lambda\sqrt{(n \cdot k)^2 - n^2 k^2}$, where $n^\mu = \sigma_\chi \delta^\mu_0$ in the matter rest frame.

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2. Photon wave function

The CP-odd domains in the chiral matter can be described by a scalar field θ whose interaction with the electromagnetic field $F^{\mu\nu}$ is governed by the Lagrangian [4,16–18]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{c_A}{4}\theta\tilde{F}_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi\,, \tag{2}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$ is the dual field tensor and c_A is the chiral anomaly coefficient [19]. A working assumption of this paper is that the field θ is spatially uniform (apart from possible thin domain walls) and a slowly varying function of time. The "free" field equations of electrodynamics in chiral electrically neutral and non-conducting matter read

$$\nabla\cdot\boldsymbol{B} = \nabla\cdot\boldsymbol{E} = 0\,, \tag{3}$$

$$\nabla\times\boldsymbol{E} = -\partial_t\boldsymbol{B}\,, \tag{4}$$

$$\nabla\times\boldsymbol{B} = \partial_t\boldsymbol{E} + \sigma_\chi\boldsymbol{B}\,, \tag{5}$$

where $\sigma_\chi = c_A\dot{\theta}$ is the chiral conductivity [11–13]. (Often $\dot{\theta}$ is denoted by μ_5 and is referred to as the axial chemical potential [14]). In the radiation gauge $A^0 = 0$ and $\nabla\cdot\boldsymbol{A} = 0$ the vector potential obeys the equation

$$-\nabla^2\boldsymbol{A} = -\partial_t^2\boldsymbol{A} + \sigma_\chi(\nabla\times\boldsymbol{A})\,. \tag{6}$$

Its solution describing a photon moving along the z-direction with energy $\omega \gg k_\perp, \sigma_\chi$ is described by the wave function

$$\boldsymbol{A}^{(0)} = \frac{1}{\sqrt{2\omega V}}\boldsymbol{e}_\lambda\, e^{ik_z z - i\omega t}\,, \quad k_z = \omega\,, \tag{7}$$

where the polarization vector satisfies $\boldsymbol{e}_\lambda\cdot\hat{\boldsymbol{z}} = 0$. V is the normalization volume. It is convenient to use the helicity basis $\boldsymbol{e}_\lambda = (\hat{\boldsymbol{x}} + i\lambda\hat{\boldsymbol{y}})/\sqrt{2}$. To determine the effect of the chiral anomaly on the photon wave function, look for a solution in the form

$$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}}(\boldsymbol{e}_\lambda\varphi + \hat{\boldsymbol{z}}\varphi')\, e^{i\omega z - i\omega t}\,, \tag{8}$$

where φ and φ' are functions of coordinates slowly varying in the longitudinal (z) direction, viz. $|\partial_z\varphi/\varphi| \ll \omega$ and $|\partial_z\varphi'/\varphi'| \ll \omega$. The two unknown functions φ and φ' are required in order to account for the change of the photon polarization direction. The gauge condition yields a constraint

$$(\boldsymbol{e}_\lambda\cdot\nabla_\perp)\varphi + \partial_z\varphi' + i\omega\varphi' \approx (\boldsymbol{e}_\lambda\cdot\nabla_\perp)\varphi + i\omega\varphi' = 0\,. \tag{9}$$

Substituting (8) into (6) one obtains

$$\boldsymbol{e}_\lambda\left(-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi\right) + \hat{\boldsymbol{z}}\left(-2i\omega\partial_z\varphi' - \nabla_\perp^2\varphi'\right) = \sigma_\chi\left(\omega\lambda\boldsymbol{e}_\lambda\varphi - \boldsymbol{e}_\lambda\times\nabla\varphi - \hat{\boldsymbol{z}}\times\nabla_\perp\varphi'\right)\,. \tag{10}$$

Taking the scalar product of this equation with $\boldsymbol{e}_\lambda^*$ and using $\boldsymbol{e}_\pm^*\cdot\boldsymbol{e}_\mp = 0$ and $\boldsymbol{e}_\pm^*\cdot\boldsymbol{e}_\pm = 1$ produces

$$-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi = \sigma_\chi(\omega\lambda\varphi - i\lambda\partial_z\varphi) + i\lambda\boldsymbol{e}_\lambda^*\cdot\nabla_\perp\varphi'\,, \tag{11}$$

where we used the identity $\boldsymbol{e}_\lambda\times\hat{\boldsymbol{z}} = i\lambda\boldsymbol{e}_\lambda$. In view of (9) one can drop the small term proportional to φ'. Neglecting also $\partial_z\varphi$ in parentheses furnishes an equation for φ:

$$-2i\omega\partial_z\varphi - \nabla_\perp^2\varphi = \sigma_\chi\omega\lambda\varphi\,. \tag{12}$$

Taking the scalar product of (10) with $\hat{\boldsymbol{z}}$ yields

$$-2i\omega\partial_z\varphi' - \nabla_\perp^2\varphi' = \sigma_\chi i\lambda(\boldsymbol{e}_\lambda\cdot\nabla)\varphi\,. \tag{13}$$

One can eliminate in (13) the term proportional to φ using the gauge condition (9). This furnishes an equation for φ' which is precisely the same as equation (12) obeyed by φ.

A solution to (12) can be written as

$$\varphi = e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\exp\left\{-i\frac{1}{2\omega}\int_0^z\left[k_\perp^2 - \sigma_\chi(z')\omega\lambda\right]dz'\right\}\,. \tag{14}$$

It follows from (9) that

$$\varphi' = -\frac{\boldsymbol{e}_\lambda\cdot\boldsymbol{k}_\perp}{\omega}\varphi\,. \tag{15}$$

Substituting (14) and (15) into (8) yields the photon wave function in the high energy approximation

$$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}}\boldsymbol{\epsilon}_\lambda\, e^{i\omega z + i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp - i\omega t} \times\exp\left\{-i\frac{1}{2\omega}\int_0^z\left[k_\perp^2 - \sigma_\chi(z')\omega\lambda\right]dz'\right\}\,, \tag{16}$$

where the polarization vector

$$\boldsymbol{\epsilon}_\lambda = \boldsymbol{e}_\lambda - \frac{\boldsymbol{e}_\lambda\cdot\boldsymbol{k}_\perp}{\omega}\hat{\boldsymbol{z}}\,. \tag{17}$$

Clearly, $\boldsymbol{\epsilon}_\lambda\cdot\boldsymbol{k} = 0$ up to the terms of order $k_\perp^2/\omega^2$ and σ_χ/ω. If the scattering process happens entirely within a single domain, then the chiral conductivity is constant. However, if a domain wall is located at, say, $z = 0$, than the chiral conductivity is different at $z < 0$ and $z > 0$. This is why a possible z-dependence of σ_χ is indicated in (16). Even though the boundary conditions on the domain wall induce a reflected wave, it can be neglected in the ultra-relativistic approximation [15,20].

It is seen in (1) that half of the infrared modes $|\boldsymbol{k}| < \lambda\sigma_\chi$ have $\mathrm{Im}\,\omega > 0$ implying exponential growth of the corresponding wave function with time. This infrared instability and its applications are discussed in many recent publications [2,8,21–36]. However, it is only tangentially related to the radiative instability discussed in this paper, even though both originate from the same dispersion relation. In particular, the infrared instability can be ignored in the ultra-relativistic limit $\omega \gg k_\perp \gg |\sigma_\chi|$ because equation

$$k_z \approx \omega - \frac{1}{2\omega}\left(k_\perp^2 - \lambda\sigma_\chi\omega\right) \tag{18}$$

has only real solutions.

3. Fermion wave function

The free fermion wave function ψ at high energy $\varepsilon \gg p_\perp, m$ can be obtained using the same procedure. Since it satisfies the Dirac equation we are looking for a solution in the form

$$\psi = \frac{1}{\sqrt{2\varepsilon V}}u(p)\phi e^{i\varepsilon z - i\varepsilon t}\,, \tag{19}$$

where $u(p)$ is a spinor describing a free fermion with momentum p and ϕ is a scalar function of coordinates. Substituting ψ into $(\partial^2 + m^2)\psi = 0$ and neglecting $\partial_z\phi$ compared to $\varepsilon\phi$ one obtains

$$2i\varepsilon\partial_z\phi + \nabla_\perp^2\phi = m^2\phi \tag{20}$$

with a solution

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