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The instability spectra of near-extremal Reissner–Nordström–de Sitter black holes

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ABSTRACT

The linearized dynamics of charged massive scalar fields in the near-extremal charged Reissner–Nordström–de Sitter (RNdS) black-hole spacetime is studied analytically. Interestingly, it is proved that the non-asymptotically flat charged black-hole-field system is characterized by *unstable* (exponentially growing in time) complex resonant modes. Using a WKB analysis in the eikonal large-mass regime $M\mu \gg 1$, we provide a remarkably compact analytical formula for the quasinormal resonant spectrum $\{\omega_n(M, Q, \Lambda, \mu, q)\}_{n=0}^{n=\infty}$ which characterizes the unstable modes of the composed RNdS-black-hole-charged-massive-scalar-field system [here $\{M, Q, \Lambda\}$ are respectively the mass, electric charge, and cosmological constant of the black-hole spacetime, and $\{\mu, q\}$ are the proper mass and charge coupling constant of the linearized field].

1. Introduction

The intriguing superradiant amplification phenomenon allows charged integer-spin fields to extract Coulomb energy and electric charge from various types of charged black holes [1–5]. As a consequence, a charged bosonic cloud surrounding a central charged black hole may grow exponentially over time if the extracted energy is not radiated fast enough to spatial infinity. Thus, the superradiant amplification mechanism imposes a non-trivial threat to the stability of charged black-hole spacetimes.

Interestingly, it has been proved in [6] that asymptotically flat charged Reissner–Nordström black-hole spacetimes are linearly stable to charged massive scalar field perturbations. On the other hand, it has recently been demonstrated numerically [7,8] that non-asymptotically flat charged Reissner–Nordström–de Sitter (RNdS) black-hole spacetimes may become unstable to perturbations of charged scalar fields whose proper frequencies lie in the bounded superradiant regime [9]

$$\frac{qQ}{r_c} < \omega < \frac{qQ}{r_+} \, , \tag{1}$$

where q is the charge coupling constant of the scalar field, $\{Q, r_+\}$ are respectively the electric charge and outer horizon radius of the central black hole, and r_c is the radius of the cosmological horizon which characterizes the black-hole spacetime.

The main goal of the present paper is to use *analytical* techniques in order to explore the physical and mathematical properties of the intriguing superradiant instability phenomenon observed in the highly interesting numerical works [7,8]. In particular, below we shall explicitly prove that the superradiant instability spectrum which characterizes the composed charged-RNdS-black-hole-charged-massive-scalar-field system can be determined analytically in the near-extremal $(r_c - r_+)/r_+ \ll 1$ regime.

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2. Description of the system

We shall analyze the quasinormal resonant modes that characterize the dynamics of a charged massive scalar field Ψ which is linearly coupled to a charged Reissner–Nordström–de Sitter black hole (see [10–12] for excellent reviews on the interesting phenomena of black-hole quasinormal resonances). The non-asymptotically flat curved black-hole spacetime is described by the line element [8,9,13]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\ , \tag{2}$$

where the radially-dependent metric function is given by the compact expression

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L^2}\ . \tag{3}$$

Here $\{M, Q, \Lambda \equiv 3/L^2\}$ are respectively the mass, electric charge [14], and the cosmological constant [15,16] of the black-hole spacetime. The radii $\{r_-, r_+, r_c\}$ of the inner (Cauchy) horizon, outer (event) horizon, and cosmological horizon which characterize the RNdS black-hole spacetime are determined by the algebraic equation

$$f(r_*) = 0 \quad \text{with} \quad * \in \{-, +, \text{c}\}\ . \tag{4}$$

The linearized dynamics of a scalar field Ψ of proper mass μ and charge coupling constant q [17] in the RNdS black-hole spacetime is governed by the Klein–Gordon wave equation [8,18]

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Psi = 0\ , \tag{5}$$

where $A_\nu = -\delta^0_\nu Q/r$ is the electromagnetic potential of the charged black-hole spacetime. It is convenient to expand the scalar field eigenfunction Ψ in the form

$$\Psi(t, r, \theta, \phi) = \int \sum_{lm} \frac{\psi_{lm}(r;\omega)}{r} Y_{lm}(\theta) e^{im\phi} e^{-i\omega t} d\omega\ , \tag{6}$$

where the integer parameters $\{l, m\}$ (which are characterized by the inequality $l \geq |m|$) denote the spherical and azimuthal harmonic indices of the charged massive scalar field eigen-modes [19]. Substituting the scalar field decomposition (6) into the Klein–Gordon wave equation (5), one obtains the ordinary differential equation

$$\frac{d^2\psi}{dy^2} + V\psi = 0 \tag{7}$$

for the radial part of the scalar eigenfunction, where the tortoise radial coordinate y is related to the areal coordinate r by the simple differential relation [20]

$$\frac{dy}{dr} = f^{-1}(r)\ . \tag{8}$$

The effective black-hole-field radial potential $V = V(r; M, Q, \Lambda, \omega, q, \mu, l)$ in the Schrödinger-like differential equation (7) is given by [8]

$$V(r) = \Big(\omega - \frac{qQ}{r}\Big)^2 - f(r)G(r)\ , \tag{9}$$

where

$$G(r) = \mu^2 + \frac{l(l+1)}{r^2} + \frac{1}{r}\frac{df}{dr}\ . \tag{10}$$

The quasinormal resonant modes, which characterize the composed RNdS-black-hole-charged-massive-scalar-field system, are determined by the Schrödinger-like ordinary differential equation (7) with the physically motivated boundary conditions of purely ingoing waves at the outer (event) horizon of the black hole and purely outgoing waves at the cosmological horizon of the spacetime [8]:

$$\psi \sim \begin{cases} e^{-i(\omega - qQ/r_+)y} & \text{for} \quad r \to r_+ \quad (y \to -\infty)\ ; \\ e^{i(\omega - qQ/r_c)y} & \text{for} \quad r \to r_c \quad (y \to \infty)\ . \end{cases} \tag{11}$$

Below we shall analyze the characteristic quasinormal resonant spectra $\{\omega_n(M, Q, \Lambda, q, \mu, l)\}_{n=0}^{n=\infty}$ of the composed near-extremal-RNdS-black-hole-charged-massive-scalar-field system. In particular, we shall consider *complex* resonant frequencies of the form

$$\omega = \omega_R - i\omega_I\ . \tag{12}$$

Taking cognizance of Eqs. (6) and (12), one realizes that resonant black-hole-field eigen-frequencies with $\omega_I < 0$ correspond to unstable (exponentially growing in time) charged field modes, whereas resonant eigen-frequencies with $\omega_I > 0$ correspond to stable (exponentially decaying) field modes.