Physics Letters B 786 (2018) 272–277

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The fundamental need for a SM Higgs and the weak gravity conjecture

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ARTICLE INFO

Article history:
Received 18 July 2018
Received in revised form 11 September 2018
Accepted 17 September 2018
Available online 19 September 2018
Editor: M. Cvetič

ABSTRACT

Compactifying the SM down to 3D or 2D one may obtain AdS vacua depending on the neutrino mass spectrum. It has been recently shown that, by insisting in the absence of these vacua, as suggested by *Weak Gravity Conjecture* (WGC) arguments, intriguing constraints on the value of the lightest neutrino mass and the 4D cosmological constant are obtained. For fixed Yukawa coupling one also obtains an upper bound on the EW scale $\langle H\rangle \lesssim \Lambda_4^{1/4}/Y_{\nu_i}$, where Λ_4 is the 4D cosmological constant and Y_{ν_i} the Yukawa coupling of the lightest (Dirac) neutrino. This bound may lead to a reassessment of the gauge hierarchy problem. In this letter, following the same line of arguments, we point out that the SM without a Higgs field would give rise to new AdS lower dimensional vacua. Absence of latter would require the very existence of the SM Higgs. Furthermore one can derive a lower bound on the Higgs vev $\langle H\rangle \gtrsim \Lambda_{QCD}$ which is required by the absence of AdS vacua in lower dimensions. The lowest number of quark/lepton generations in which this need for a Higgs applies is three, giving a justification for family replication. We also reassess the connection between the EW scale, neutrino masses and the c.c. in this approach. The EW fine-tuning is here related to the proximity between the c.c. scale $\Lambda_4^{1/4}$ and the lightest neutrino mass m_{ν_i} by the expression $\frac{\Delta H}{H} \lesssim \frac{(a\Lambda_4^{1/4}-m_{\nu_i})}{m_{\nu_i}}$. We emphasize that all the above results rely on the assumption of the stability of the AdS SM vacua found.

1. Introduction

It is a frustrating fact how poor our present understanding of the origin of the different fundamental mass scales in Particle Physics is. Simplifying a bit, there are essentially three regions of scales in fundamental physics. There is a deep-infrared region in which there are only two fundamental massless particles, photon and graviton with the three neutrinos with masses in the region $m_{\nu_i} \lesssim 10^{-3}$–$10^{-1}$ eV, where one of the neutrinos could even be massless. Interestingly, this is also very close to the scale of the observed cosmological constant $\Lambda_4 = (2.25 \times 10^{-3}\ \text{eV})^4$.[1] The second region is that of the masses of most elementary particles which are around 10^{-3}–10^2 GeV. These masses are dictated both by the value of the QCD condensate $\Lambda_{QCD} \simeq 10^{-1}$ GeV and the Higgs vev $\langle H^0\rangle = 246$ GeV. Finally there is the Planck scale and presumably a unification/string scale somewhat below. We would like, of course, to understand why the scales are what they are and what is the information that this distribution of scales is giving us concerning the fundamental theory. In particular it is difficult to understand why Λ_4 and the EW scale are so small compared to the fundamental scales of gravity and unification. Also, the proximity of neutrino masses to $\Lambda^{1/4}$ as well as the (relative) proximity of Λ_{QCD} to the EW scale could be just coincidences or could be telling us something about the underlying theory.

A natural question is whether all these scales are independent or whether they are related or constrained within a more fundamental theory including quantum gravity coupled to the SM physics. Recently it has been pointed out that quantum gravity constraints could have an impact on Particle Physics [1–3]. The origin of these constraints is based on the Weak Gravity Conjecture (WGC) [4,5], see [6] for a review and [7–9] for some recent references. A *sharpened* variation of the WGC was proposed by Ooguri and Vafa in [1] which states that a non-SUSY Anti-de Sitter stable vacuum cannot be embedded into a consistent theory of quantum gravity (see also [10]). This general statement, together with an assumption of background independence, may be applied to the Standard Model (SM) itself [1] implying that no compactification of the SM to lower dimensions should lead to a stable AdS vac-

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[1] We are assuming here that the origin of dark energy is a 4D cosmological constant.

https://doi.org/10.1016/j.physletb.2018.09.034

uum, if indeed the SM is to be consistently coupled to quantum gravity.

Interestingly, the exercise of compactifying the SM down to 3D and 2D was already done by Arkani-Hamed et al. [11,12] a long time ago, with a totally different motivation. They found that there may be 3D and 2D SM AdS vacua depending on the values of neutrino masses, via a radion potential induced by the Casimir effect. By assuming those results OV claimed that their sharpened WGC would imply the inconsistency of Majorana neutrino masses. In [2] a thorough analysis of this question was presented. It was further found that the 4D cosmological constant is bounded below by the value of the lightest neutrino mass, providing an explanation for the apparent proximity of both quantities. Furthermore, it was shown that the same bound, for fixed Λ_4 and Yukawa couplings, induces an upper bound on the Higgs vev, giving an explanation for the stability of the Higgs potential of the SM [2,3]. This bound implies that the gauge hierarchy stability may be a consequence of quantum gravity constraints for fixed values of Λ_4 and the neutrino mass. We reassess this issue at the end of this note.

Before proceeding let us emphasize that the stability of the dangerous AdS vacua found from SM compactifications is a strong assumption. Indeed, working with an effective field theory it is impossible to know whether the theory has some unknown instabilities in the UV. Undoubtedly, the embedding of the SM (or an extension of it) in a UV complete theory like string theory would make arguments about stability stronger. Lacking this, all the constraints obtained in this letter rely on the assumption that the AdS vacua found in the effective field theory are stable. The fact that this, admittedly, bold assumption leads to a number of intriguing results and constraints, makes this assumption worth exploring.

Apart from the possible mentioned instabilities in the UV, there may be still some sources of instabilities at the effective field theory level. That is the case of the massless scalars associated to the SM Wilson lines, which may give rise to runaway scalar directions [13]. Nevertheless, it turns out that those scalars are projected out from the spectrum in certain vacua, like toroidal Z_N 2D Standard Model vacua [14], so that again one can recover the same bounds of the circle or toroidal compactifications. However, one also finds within this class of vacua examples in which the minimal SM necessarily develops AdS stable vacua, irrespective of the value of neutrino masses nor any other SM parameter [14]. Thus, if these WGC arguments are correct, and the AdS vacua are indeed stable, the SM by itself would be in the *swampland*. Certainly, this does not mean that the observed SM is incompatible with quantum gravity, since modifications BSM above the EW scale can render those AdS vacua unstable. Indeed this is what happens in a SUSY completion of the SM like the MSSM with appropriate discrete gauge symmetries. This is true in particular provided the $U(1)_{B-L}$ symmetry (or a discrete subgroup of it) is gauged at some scale [14]. In connection to this, note that the minimal 4D SM seems to have a second Higgs AdS vacuum at around $\langle H\rangle \simeq 10^{10-12}$ GeV. Thus, if this second AdS minimum exists and is stable, it would need some modification (like SUSY) at higher energies anyway.

In the present note we obtain new constraints on the Higgs vev by imposing the absence of lower dimensional SM AdS vacua. Of course, these constraints rely on the aforementioned assumptions that the minima found are indeed stable in the UV and that the SM action that we study is completed at high energies in such a way that the AdS vacua of [14] and the one already existing in 4D are either absent or unstable.

The new bounds we find here are independent from the neutrino bounds. We find that in order to avoid AdS vacua:

- A Higgs with non-vanishing vev and Yukawa couplings must exist.
- There is a lower bound on the Higgs vevs for fixed Yukawa couplings $\langle H\rangle \gtrsim \Lambda_{\rm QCD}$.
- The minimum number of generations for which the existence of a Higgs is mandatory is three.

In deriving these conclusions we are setting fixed the values of the dimensionless couplings (Yukawa and gauge couplings) as well as the measured value of the cosmological constant. We discuss the combination of the above lower bound with the upper bound coming from the absence of neutrino generated AdS vacua. We also rephrase the upper bound on neutrino masses as a constraint relating the EW fine-tuning with the proximity between the c.c. and the neutrino mass scale.

2. A world with no Higgs is in the swampland

Let us consider first the fermion and gauge boson content of the SM (plus the graviton) with n_g quark/lepton generations. In the absence of the Higgs, the theory has an (approximate) $U(2n_g)_L \times U(2n_g)_R$ accidental global symmetry in the quark sector. This symmetry is spontaneously broken by the QCD condensate of the quarks down to $U(2n_g)_{L+R}$, generating a total of $4n_g^2$ massless Goldstone bosons. Three of them become massive by combining with the $W^\pm$ and Z bosons, which acquire masses given by:

$$m_W = \sqrt{n_g}\,\frac{g f_\pi}{2} \tag{2.1}$$

$$m_Z = m_W / \cos\theta_{\rm W}, \tag{2.2}$$

where n_g is the number of generations and f_π is the Goldstone boson decay constant. In the physical QCD case with $n_g = 3$ the latter is given for the pion by $f_\pi = 93$ MeV. More generally one has for the Goldstone boson decay constants $f_G \simeq \Lambda_{\rm QCD}$. One more Goldstone boson is expected to become massive due to the QCD anomaly, so that below the $\Lambda_{\rm QCD}$ scale we are left with a total of $4(n_g^2-1)$ Goldstone bosons. In fact all of these are actually pseudo-Goldstone bosons which get mass from electroweak corrections, see the discussion below. These masses appear at the one-loop level, so they are below the EW gauge bosons masses. We will take these masses into account in the numerical evaluations but we follow here the discussion as if they were actually massless to illustrate the counting of degrees of freedom which is relevant for the Casimir potential. In addition to the pseudo-Goldstones there are 4 more bosonic degrees of freedom from the massless photon and graviton, so that the number of light bosonic degrees of freedom below the QCD scale is $N_B = 4n_g^2$. The total fermionic minus bosonic degrees of freedom below $\Lambda_{\rm QCD}$ is then

$$(N_F - N_B)^{<\Lambda_{\rm QCD}} = 8n_g - 4n_g^2 = 4n_g(2-n_g)\,, \tag{2.3}$$

where the fermionic degrees of freedom correspond to charged leptons and (Dirac) neutrinos (we are taking Dirac rather than Majorana neutrinos because the latter lead necessarily to AdS vacua, as shown in [2]). Note that above $\Lambda_{\rm QCD}$ one has leptons and unconfined quarks and then one rather has

$$(N_F - N_B)^{>\Lambda_{\rm QCD}} = 32n_g - 24 - 2\,, \tag{2.4}$$

where the 24 comes from the SM gauge bosons and the 2 from the graviton. The value of $(N_F - N_B)$ is crucial since the Casimir potential of the radion upon compactification of the SM down to 3D or 2D depends linearly on it. Since above the QCD transition it is always positive, an AdS minimum will develop if it is negative