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# Casimir effect for mixed fields

M. Blasone [a,b], G.G. Luciano [a,b], L. Petruzziello [a,b,*], L. Smaldone [a,b]

[a] Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, Italy
[b] INFN, Gruppo collegato di Salerno, Italy

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ABSTRACT

We analyze the Casimir effect for a flavor doublet of mixed scalar fields confined inside a one-dimensional finite region. In the framework of the unitary inequivalence between mass and flavor representations in quantum field theory, we employ two alternative approaches to derive the Casimir force: in the first case, the zero-point energy is evaluated for the vacuum of fields with definite mass, then similar calculations are performed for the vacuum of fields with definite flavor. We find that signatures of mixing only appear in the latter context, showing the result to be independent of the mixing parameters in the former.



## 1. Introduction

The concept of vacuum in quantum field theory (QFT) is as fascinating as puzzling. In several situations from both particle physics and condensed matter, the non-trivial condensate structure of the vacuum is crucial to explain a variety of both theoretical and observable phenomena [1–4]. In this connection, one of the most eloquent examples is provided by the Casimir effect [5], which occurs whenever a quantum field is enclosed in a finite region; such a confinement gives rise to a net attractive force between the boundaries, the entity of which is closely related to the nature of the vacuum itself [6].

In line with these findings, in Refs. [7,8] it was shown that vacuum also plays a central rôle within the framework of flavor mixing and oscillations in QFT. In Refs. [7], in particular, it was found that the vacuum for fields with definite mass (mass vacuum) is *unitarily inequivalent* [9,10] to the one for fields with definite flavor (flavor vacuum), as they are related by a non-trivial Bogoliubov transformation. In light of this, it is reasonable to expect that vacuum effects in the context of QFT mixing may, in principle, depend on which of these states represents the physical vacuum. This is indeed a matter of open debate [11]: an interesting test bench in this sense has been recently provided by the analysis of the weak decay of accelerated protons (inverse $\beta$-decay) with mixed neutrinos [12–14].

Led by these considerations, here we analyze the Casimir effect for a system of two mixed scalar fields, showing that the force is sensitive to the choice of the vacuum state. In particular, we find that the result obtained using the flavor vacuum exhibits corrections that explicitly depend on the mixing angle and the mass difference of fields, in contrast with the case of the mass vacuum.

We remark that, although limited to scalar fields in $1+1$ dimensions, our analysis contains all the essential features of the problem, thus giving general validity to our results. We also stress that the local nature of the Casimir force prevents our calculations from being affected by the choice of a particular regularization scheme. Such a characteristic is not present in other contexts, where effects of the flavor vacuum have been studied [15].

The paper is organized as follows: Sec. 2 is devoted to briefly review the derivation of the Casimir force for a massive scalar field in $1+1$ dimensions. In Sec. 3, we analyze how the standard expression gets modified in the presence of mixed fields by performing calculations on both mass and flavor vacua. Sec. 4 contains conclusions and an outlook for future developments. Throughout the paper, we use natural units and the metric in the conventional timelike signature.

## 2. Casimir effect for a massive scalar field

Let us start by deriving the Casimir force for a massive charged scalar field $\hat{\phi}$ in $1+1$ dimensions (to this aim, we basically follow

* Corresponding author.
*E-mail addresses:* blasone@sa.infn.it (M. Blasone), gluciano@sa.infn.it (G.G. Luciano), lpetruzziello@na.infn.it (L. Petruzziello), lsmaldone@sa.infn.it (L. Smaldone).

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the treatment of Ref. [16]). In this framework, the free Lagrangian density $\hat{\mathcal{L}}$ takes the form[1]

$$\hat{\mathcal{L}} = \partial_\mu \hat{\phi}^\dagger \partial^\mu \hat{\phi} - m^2 \hat{\phi}^\dagger \hat{\phi}\,, \tag{1}$$

where $m$ is the mass of the field.

The Dirichlet boundary conditions imposed by the presence of the Casimir plates read

$$\hat{\phi}(t,0) = \hat{\phi}(t,L) = 0\,, \tag{2}$$

with $L$ being the distance between the two confining surfaces. These constraints only allow modes with momentum $k_n = \pi n/L$ to give a non-vanishing contribution to the field expansion, yielding

$$\hat{\phi}(t,x) = \frac{1}{\sqrt{2L}} \sum_{n=1}^{\infty} \frac{\sin k_n x}{\sqrt{\omega_n}} \left[\hat{a}_n\, e^{-i\omega_n t} + \hat{b}_n^\dagger\, e^{i\omega_n t}\right], \tag{3}$$

where $n = 1, 2, \ldots$ and $\omega_n = \sqrt{k_n^2 + m^2}$. Here $\hat{a}_n$ ($\hat{b}_n^\dagger$) are the usual annihilation (creation) operators of a particle (antiparticle) with momentum $k_n$ and frequency $\omega_n$. They are assumed to satisfy the canonical bosonic algebra

$$\left[\hat{a}_n, \hat{a}_{n'}^\dagger\right] = \left[\hat{b}_n, \hat{b}_{n'}^\dagger\right] = \delta_{nn'}\,, \qquad \forall n, n', \tag{4}$$

with all other commutators vanishing. The vacuum state is defined by

$$\hat{a}_n|0\rangle = \hat{b}_n|0\rangle = 0, \quad \forall n\,. \tag{5}$$

In order to compute the Casimir force, let us now evaluate the zero-point energy density of the field as

$$\varepsilon_0 = \langle 0|\hat{T}_{00}|0\rangle\,, \tag{6}$$

where $\hat{T}_{\mu\nu}$ is the stress-energy tensor derived from the Lagrangian density Eq. (1) [17]. A straightforward calculation leads to

$$\varepsilon_0 = \frac{1}{2L} \sum_{n=1}^{\infty} \omega_n\,. \tag{7}$$

Using the standard definition of Casimir force [6,18]

$$F_0 = -\frac{\partial}{\partial L}(L\,\varepsilon_0)\,, \tag{8}$$

and exploiting a suitable renormalization scheme [16], we finally obtain the following finite expression for the net force between the plates:

$$F = -\frac{m^2}{\pi} \sum_{n=1}^{\infty} \left[K_2(2mLn) - \frac{K_1(2mLn)}{2mLn}\right], \tag{9}$$

where $K_\nu(x)$ is the modified Bessel function of the second kind [19]. Notice that, in the limit $m \to 0$, Eq. (9) correctly reproduces the more familiar expression of the Casimir force for a massless field [6,16,18].

[1] To simplify the notation, we shall omit the $(t, x)$-dependence of the field when unnecessary.

## 3. Casimir effect for mixed fields

Let us now generalize the above formalism to the context of field mixing. For this purpose, consider the following Lagrangian density describing two charged scalar fields with a mixed mass term [8]:

$$\hat{\mathcal{L}} = \sum_{\sigma=A,B} \left(\partial_\mu \hat{\phi}_\sigma^\dagger \partial^\mu \hat{\phi}_\sigma - m_\sigma^2 \hat{\phi}_\sigma^\dagger \hat{\phi}_\sigma\right) - m_{AB}^2 \left(\hat{\phi}_A^\dagger \hat{\phi}_B + \hat{\phi}_B^\dagger \hat{\phi}_A\right), \tag{10}$$

where $\hat{\phi}_\sigma$ ($\sigma = A, B$) are the fields with definite flavor $\sigma$.

It is a trivial matter to check that the mixing transformations

$$\begin{pmatrix} \hat{\phi}_A \\ \hat{\phi}_B \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{pmatrix}, \tag{11}$$

allow to recast the quadratic form Eq. (10) into a diagonal Lagrangian density for two free charged scalar fields $\hat{\phi}_j$ ($j = 1, 2$) with mass $m_j$:

$$\hat{\mathcal{L}} = \sum_{j=1,2} \left(\partial_\mu \hat{\phi}_j^\dagger \partial^\mu \hat{\phi}_j - m_j^2 \hat{\phi}_j^\dagger \hat{\phi}_j\right), \tag{12}$$

where the two sets of mass parameters $m_\sigma$ and $m_j$ are related by

$$m_A^2 = \cos^2\theta\, m_1^2 + \sin^2\theta\, m_2^2\,, \tag{13}$$

$$m_B^2 = \sin^2\theta\, m_1^2 + \cos^2\theta\, m_2^2\,, \tag{14}$$

and $m_{AB}^2$ in Eq. (10) is given by $m_{AB}^2 = (m_2^2 - m_1^2)\sin\theta\cos\theta$. The mixing angle $\theta$ is defined as $\tan 2\theta = 2m_{AB}^2/\left(m_B^2 - m_A^2\right)$.

Note that each of the two fields $\hat{\phi}_j$ ($j = 1, 2$) in Eq. (11) can be expanded as in Eq. (3). Thus, according to Eq. (5), one can define the vacuum for fields with definite mass (*mass vacuum*) as

$$\hat{a}_{n,j}|0\rangle_{1,2} = \hat{b}_{n,j}|0\rangle_{1,2} = 0\,, \qquad \forall n, \quad j = 1, 2\,. \tag{15}$$

To derive the corresponding relation for fields with definite flavor, it is worth rewriting Eq. (11) in terms of the mixing generator $K_{\theta,\mu}(t)$ [20] as:

$$\hat{\phi}_\chi(t,x) = K_{\theta,\mu}^{-1}(t)\, \hat{\phi}_l(t,x)\, K_{\theta,\mu}(t)\,,$$
$$(\chi, l) = (A,1), (B,2)\,, \tag{16}$$

where $K_{\theta,\mu}(t) = G_\theta(t)\, I_\mu(t)$, with

$$G_\theta(t) = \exp\Big[-i\theta \int_0^L dx \left(\hat{\pi}_1(t,x)\hat{\phi}_2(t,x) + \hat{\phi}_2^\dagger(t,x)\hat{\pi}_1^\dagger(t,x)\right.$$
$$\left. - \hat{\pi}_2(t,x)\hat{\phi}_1(t,x) - \hat{\phi}_1^\dagger(t,x)\hat{\pi}_2^\dagger(t,x)\right)\Big], \tag{17}$$

and

$$I_\mu(t) = \exp\left[\sum_{n=1}^{\infty} \sum_{\sigma,j} \xi_{\sigma,j}^n \left(a_{n,\sigma}^\dagger(t) b_{n,\sigma}^\dagger(t) - b_{n,\sigma}(t) a_{n,\sigma}(t)\right)\right]. \tag{18}$$

Here $\hat{\pi}_j \equiv \partial_t \phi_j^\dagger$ ($j = 1, 2$) is the canonical momentum conjugate to the field $\hat{\phi}_j$, $\xi_{\sigma,j}^n \equiv \frac{1}{2}\log\left(\frac{\omega_{n,\sigma}}{\omega_{n,j}}\right)$ and $\omega_{n,\sigma} = \sqrt{k_n^2 + \mu_\sigma^2}$ ($\sigma = A, B$).

For $\mu_A = m_1$ and $\mu_B = m_2$, one can easily check that $I_\mu(t) = \mathbb{1}$, and the field expansions for definite flavor fields read