

# Generalized two-field $\alpha$ -attractor models from geometrically finite hyperbolic surfaces

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## Abstract

We consider four-dimensional gravity coupled to a non-linear sigma model whose scalar manifold is a non-compact geometrically finite surface  $\Sigma$  endowed with a Riemannian metric of constant negative curvature. When the space-time is an FLRW universe, such theories produce a very wide generalization of two-field  $\alpha$ -attractor models, being parameterized by a positive constant  $\alpha$ , by the choice of a finitely-generated surface group  $\Gamma \subset \mathrm{PSL}(2, \mathbb{R})$  (which is isomorphic with the fundamental group of  $\Sigma$ ) and by the choice of a scalar potential defined on  $\Sigma$ . The traditional two-field  $\alpha$ -attractor models arise when  $\Gamma$  is the trivial group, in which case  $\Sigma$  is the Poincaré disk. We give a general prescription for the study of such models through uniformization in the so-called “non-elementary” case and discuss some of their qualitative features in the gradient flow approximation, which we relate to Morse theory. We also discuss some aspects of the SRST approximation in these models, showing that it is generally not well-suited for studying dynamics near cusp ends. When  $\Sigma$  is non-compact and the scalar potential is “well-behaved” at the ends, we show that, in the *naïve* local one-field truncation, our generalized models have the same universal behavior as ordinary one-field  $\alpha$ -attractors if inflation happens near any of the ends of  $\Sigma$  where the extended potential has a local maximum, for trajectories which are well approximated by non-canonically parameterized geodesics near the ends; we also discuss spiral trajectories near the ends. Generalized two field  $\alpha$ -attractors illustrate interesting consequences of nonlinear sigma models whose scalar manifold is not simply connected. They provide a large class of tractable cosmological models with non-trivial topology of the scalar field space.

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## 0. Introduction

Inflation in the early universe can be described reasonably well by so-called  $\alpha$ -attractor models [1–6]. In their two-field version (see, for example, [5,6]), such models arise from cosmological solutions of four-dimensional gravity coupled to a nonlinear sigma model whose scalar manifold  $\Sigma$  (i.e. the target manifold of the system of two real scalar fields) is the open unit disk endowed with its unique complete metric  $\mathcal{G}$  (which determines the kinetic energy term of the scalar fields) of constant Gaussian curvature  $K$  equal to  $-\frac{1}{3\alpha}$ , where  $\alpha$  is a positive constant. The “universal” behavior of such models in the radial one-field truncation close to the conformal boundary of the unit disk is a consequence of the hyperbolic character of  $\mathcal{G}$  [4–6].

While ordinary one-field<sup>2</sup> models suffice to explain current cosmological data [7], there are at least a few good reasons to study the inflationary and post-inflationary dynamics of two-field (and of more general multi-field) models, which form a subject of active and continued interest [8–24]. First, it is possible that higher precision observations in the medium future may detect deviations from one-field model predictions. Second, it is considerably easier to produce multi-field models in fundamental theories of gravity (such as string theory) than it is to produce one-field models. Third, multi-field models are of theoretical interest in themselves. In particular, it is well-known that such models display behavior which is qualitatively new with respect to that of one-field models; this happens due to the higher-dimensionality of the target manifold of the system of real scalar fields.

In this paper, we initiate a systematic study of two-field cosmological models whose target manifold is an arbitrary borderless, connected, oriented and non-compact two-dimensional smooth manifold  $\Sigma$  endowed with a complete Riemannian metric  $\mathcal{G}$  of constant negative curvature. Similar to the case of ordinary two-field  $\alpha$ -attractor models, the Gaussian curvature  $K$  of  $\mathcal{G}$  can be parameterized by a positive constant  $\alpha$  which is defined through the relation  $K = -\frac{1}{3\alpha}$ . Since the open unit disk endowed with its unique complete metric of constant and fixed negative Gaussian curvature  $K$  provides the simplest example of such a Riemannian two-manifold, the cosmological models considered in this paper can form an extremely wide generalization of ordinary two-field  $\alpha$ -attractors, so we shall call them *two-field generalized  $\alpha$ -attractor models*.

Writing  $\mathcal{G} = 3\alpha G$  produces a Riemannian metric  $G$  on  $\Sigma$  whose Gaussian curvature is constant and equal to  $-1$ . Thus  $(\Sigma, G)$  is an (oriented, connected, non-compact and borderless) *hyperbolic surface* and the two-field cosmological model defined by  $(\Sigma, \mathcal{G})$  is equivalently parameterized by the real positive number  $\alpha$  and by  $(\Sigma, G)$ . The geometry and topology of non-compact hyperbolic surfaces are extremely rich. For example, such a surface can have infinite genus as well as a (countable) infinity of Freudenthal ends [25–27]; a simple example of this phenomenon is provided by the surface  $\Sigma = \mathbb{C} \setminus \mathbb{Z}$ , which has infinite genus and whose set of ends can be identified with the set of integer numbers. A full topological classification of oriented, borderless, connected and non-compact surfaces is provided by the so-called Kerékjártó–Stoilow model (see [28]). By the uniformization theorem of Poincaré and Koebe (see [29] for a modern account), a hyperbolic surface  $(\Sigma, G)$  is parameterized up to isometry by the choice of the con-

<sup>2</sup> We count the number of *real* scalar fields present in the model.

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