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Thermodynamic limit and twisted boundary energy of the XXZ spin chain with antiperiodic boundary condition

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Abstract

We investigate the thermodynamic limit of the inhomogeneous $T - Q$ relation of the antiferromagnetic XXZ spin chain with antiperiodic boundary condition. It is shown that the contribution of the inhomogeneous term for the ground state can be neglected when the system-size N tends to infinity, which enables us to reduce the inhomogeneous Bethe ansatz equations (BAEs) to the homogeneous ones. Then the quantum numbers at the ground states are obtained, by which the system with arbitrary size can be studied. We also calculate the twisted boundary energy of the system.

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1. Introduction

The XXZ spin chain with the antiperiodic boundary condition (or the twisted boundary condition) is a very interesting quantum system [1–4]. By using the Jordan–Wigner transformation, the model can describe a p-wave Josephson junction embedded in a spinless Luttinger liquid [5–7]. Although there exists a twisted bound at the boundary which breaks the usual $U(1)$-symmetry of the bulk system (or the closed chain case) [8], it can be proved that the system is still integrable. By using the off-diagonal Bethe ansatz (ODBA) method [9–11], the exact solution of the model was obtained [9], which is described by an inhomogeneous $T-Q$ relation (cf. the ordinary homogeneous $T-Q$ one [12,13]). Such an inhomogeneous $T-Q$ relation has played a universal role to describe the eigenvalue of the transfer matrix for quantum integrable systems [8]. However, due to the fact that Bethe roots should satisfy the inhomogeneous Bethe ansatz equations (BAEs), it is hard to study the thermodynamic properties [14] of the corresponding systems [15–17].

Based on an intelligent trick, the thermodynamic limit of the spin-$\frac{1}{2}$ XXZ chain with the generic off-diagonal boundary terms in the gapless region (i.e., the anisotropy parameter η in (2.1) below being an imaginary number) was succeeded in obtaining [18]. The most important observation in the paper is that the contribution of the inhomogeneous term for the ground state, in the gapless region, can be neglected when the system-size N tends to infinity. Such a fact has been confirmed recently by the studies of other integrable models [19–22] whose eigenvalue of the transfer matrix is given in terms of the inhomogeneous $T-Q$ relation.

In this paper, we propose a method to study the thermodynamic limit of the XXZ spin chain with the twisted boundary condition at the antiferromagnetic region (i.e., η being a real number). We first study the contribution of the inhomogeneous term with finite system-size N. We find that the contribution of the inhomogeneous term in the associated $T-Q$ relation to the ground state energy can be neglected when the system-size N tends to infinity. Because we consider the massive region of the system, the ground state energy with even N and that with odd N are different. The value of energy difference is proportional to the energy of one bond. We also check our results by using the density matrix renormalization group (DMRG) method [23,24], which leads to that the numerical results and the analytic one are consistent with each other very well. As a consequence, we obtain the twisted boundary energy of the model.

The paper is organized as follows. In the next section, the model and the associated ODBA solutions are introduced. In section 3, we study the finite-size effects of contribution of the inhomogeneous term in the $T-Q$ relation for the ground state. The thermodynamic limit of the XXZ spin chain with antiperiodic and with periodic boundary conditions are discussed in section 4 and section 5, respectively. The twisted boundary energy is given in Section 6. Section 7 is the concluding remarks and discussions. Some supporting detailed calculations are given in Appendices A and B.

2. The model and its ODBA solution

The spin-$\frac{1}{2}$ XXZ quantum chain is described by the Hamiltonian

$$H=\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x+\sigma_j^y\sigma_{j+1}^y+\cosh\eta\,\sigma_j^z\sigma_{j+1}^z\right], \tag{2.1}$$