Nonlinear Analysis: Real World Applications 46 (2019) 43–57

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Asymptotic stability of planar rarefaction wave to 3D radiative hydrodynamics

Bingkang Huang[a], Lan Zhang[b,c,*]

[a] School of Mathematics, Hefei University of Technology, Hefei 230009, China
[b] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
[c] Computational Science Hubei Key Laboratory, Wuhan University, Wuhan 430072, China

ARTICLE INFO

Article history:
Received 20 May 2018
Accepted 10 September 2018
Available online xxxx

Keywords:
3d radiative hydrodynamics
Rarefaction waves
Asymptotic stability

ABSTRACT

We are concerned with the large-time behavior of the planar rarefaction wave for the 3d radiative hydrodynamics. We show the nonlinear stability of the planar rarefaction wave in two cases. For the one, the viscosity coefficient μ and transport coefficient κ are positive constant and the proof is motivated by Li et al. (2018). For another, we neglect the transport coefficient κ. It is crucial to use the symmetrization of quasi-linear form of the perturbation system (4.42).

1. Introduction and main result

We consider the large time behavior of global solutions to the Cauchy problem of the 3D radiative hydrodynamics:

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) &= 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla p &= \mathrm{div}\mathscr{T}, \\ (\rho E)_t + \mathrm{div}(\rho E u + p u) + \mathrm{div} q &= \kappa \Delta\theta + \mathrm{div}(u\mathscr{T}), \\ -\nabla \mathrm{div} q + a q + b\nabla\theta^4 &= 0. \end{cases} \tag{1.1}$$

here, the unknowns are $\rho(t,x)$, $u(t,x)$ and $\theta(t,x)$, for $t \geq 0$, $x = (x_1, x_2, x_3) \in \Omega \subset \mathbb{R}^3$, denoting respectively the fluid density, velocity and absolute temperature, $E = e + \frac{1}{2}|u|^2$ is the specific total energy, $\mathscr{T} = 2\mu\mathbb{D}(u) + \lambda \mathrm{div} u \mathbb{I}$ is the viscous stress tensor where $\mathbb{D}(u) = \frac{\nabla u + (\nabla u)^t}{2}$ is the deformation tensor. The viscosity coefficient μ and the heat conductivity coefficient κ are constants. Here we focus on the ideal polytropic gas, that is, the pressure p and the specific internal energy e are given by the constitutive relations

$$p = R\rho\theta, \quad e = c_v\theta + \mathrm{const.}, \tag{1.2}$$

where $R > 0$ is the gas constant and the specific heat $c_v = R/(\gamma - 1)$ with $\gamma > 1$ being the adiabatic exponent. In this paper, we consider the astrophysical flows in an infinite long flat nozzle domain $\Omega := \mathbb{R} \times \mathbb{T}^2$.

* Corresponding author.
E-mail addresses: bkhuang92@hotmail.com (B. Huang), zhang_lan@whu.edu.cn (L. Zhang).

https://doi.org/10.1016/j.nonrwa.2018.09.003
1468-1218/

The system (1.1) is supplemented with the initial data

$$(v,u,\theta)|_{t=0}=(v_0,u_0,\theta_0), \tag{1.3}$$

which is assumed to satisfy the far-field condition:

$$\lim_{x_1\to\pm\infty}(v_0,u_0,\theta_0)(x)=(v_\pm,u_\pm,\theta_\pm), \tag{1.4}$$

where $v_\pm>0$, $u_\pm$ and $\theta_\pm>0$ are given constants, $u_\pm=(u_{1\pm},0,0)^t$ and the periodic boundary conditions are imposed on $(x_2,x_3)\in\mathbb{T}^2$. The system (1.1)–(1.3) describes a situation where the gas is not in thermodynamical equilibrium with radiation. There are many literatures which consider the radiation hydrodynamics from the mathematical and physical point of view, such as, [1–7].

In one dimensional case, it is well-known that the large time behavior of the global solutions $(\rho(t,x_1),u(t,x_1),\theta(t,x_1))$ is determined by the structure of the unique global entropy solution $(\rho^r(\frac{x}{t}),u^r(\frac{x}{t}),\theta^r(\frac{x}{t}))$ of the following 1d compressible Euler equations:

$$\begin{cases} \rho_t+(\rho u_1)_{x_1} &=0,\\ (\rho u_1)_t+(\rho u_1^2+p)_{x_1} &=0,\\ (\rho E)_t+(\rho E u_1+pu_1)_{x_1} &=0. \end{cases} \tag{1.5}$$

with the Riemann initial data

$$(\rho,u_1,\theta)(0,x_1)=(\rho_0^r,u_{10}^r,\theta_0^r)(x_1)=\begin{cases}(\rho_-,u_{1-},\theta_-), & x_1<0\\ (\rho_+,u_{1+},\theta_+), & x_1>0\end{cases}$$

The solutions consist of three wave patterns called shock wave, rarefaction wave and linearly degenerate wave and their superpositions, called by Riemann solutions, and govern both the local and large time asymptotic behavior of general solutions of the system (1.5). It is of great importance and interest to study the large-time behavior of the viscous version of these basic wave patterns and their superpositions to the 3d radiative hydrodynamics (1.1).

The problem we want to study is the large time behavior of the planar rarefaction wave of the 3d radiative hydrodynamics (1.1). The stability of the rarefaction wave for viscous regularization of hyperbolic systems has been well studied. For the small initial perturbation, the authors proved the stability for isentropic flow in [8,9]. For the full compressible Navier–Stokes equation, the corresponding result has been studied in [10–12]. Recently, in [13,14] the authors proved the stability of the rarefaction wave under large initial perturbation.

For a simplified model of radiating gas dynamics, the authors in [15,16] proved the stability of the rarefaction wave. This model can be regarded as a Burgers-type equation coupled with a linear elliptic equation for the radiation. Since the equation for gas is scalar, it is available to prove the stability of the planar rarefaction wave for the simplified model. The stability of the planar rarefaction wave for scalar conservation law has been proved in [17–19]. Then the result was generalized to the two dimensional and $n(n\geq 3)$ dimensional simplified model of radiating gas dynamics in [20,21]. Moreover a lot of works considered the full system of radiative hydrodynamic equations. The stability of rarefaction wave is studied in [6] and the stability of viscous contact wave is considered in [22,23]. The asymptotic stability of their combination is proved in [24,25]. Recently, Li et al. [26] proved the time-asymptotic stability toward planar rarefaction wave for three-dimensional full compressible Navier–Stokes equations in a finite long flat nozzle domain in $\mathbb{R}\times\mathbb{T}^2$. Inspired by their result, we wish to deal with the full system of 3d hydrodynamic equations coupled with a nonlinear elliptic equation.

We first introduce you the planar rarefaction wave. The rarefaction wave (ρ,u_1,θ) is characterized by

$$\begin{cases}\lambda_i(\rho,u_1,s)=u_1^r+(-1)^{\frac{i+1}{2}}\sqrt{p_\rho(\rho,s)}, \quad i=1,3\\ \Sigma_i^j=u_1+(-1)^{\frac{i-1}{2}}\int^\rho\frac{\sqrt{p_z(z,s)}}{z}, \quad i=1,3,\quad j=1,2\end{cases}$$