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On local strong solutions to the three-dimensional nonhomogeneous Navier–Stokes equations with density-dependent viscosity and vacuum

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ABSTRACT

We study the three-dimensional nonhomogeneous Navier–Stokes equations with density-dependent viscosity and vacuum on $\Omega \subset \mathbb{R}^3$, which is either a bounded domain or a usual unbounded one such as the whole space $\mathbb{R}^3$ and an exterior one. In particular, the initial density can have compact support when Ω is unbounded. For initial data without additional compatibility conditions, we prove that there exists a unique local strong solution to the initial and initial boundary value problems. Moreover, the continuous of strong solutions on the initial data is derived under an additional compatibility condition.

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1. Introduction and main results

We consider the three-dimensional nonhomogeneous Navier–Stokes equations which read as follows:

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) - \operatorname{div}(2\mu(\rho) d) + \nabla P = 0, \\ \operatorname{div} u = 0, \end{cases} \tag{1.1}$$

where $t \geq 0$ is time, $x = (x_1, x_2, x_3) \in \Omega \subset \mathbb{R}^3$ is the spatial coordinate, $\rho = \rho(x,t)$, $u = (u^1, u^2, u^3)(x,t)$ and $P = P(x,t)$ denote the density, velocity and pressure of the fluid, respectively;

$$d = \frac{1}{2}(\nabla u + \nabla^T u)$$

is the deformation tensor; the viscosity $\mu = \mu(\rho)$ satisfies the following hypothesis:

$$\mu \in C^1[0,\infty) \quad \text{and} \quad \mu > 0 \quad \text{on} \quad [0,\infty). \tag{1.2}$$

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Let $\Omega \subset \mathbb{R}^3$ be a connected smooth domain, either bounded or unbounded. More precisely, the domain Ω could be a bounded domain, an exterior one, and the whole three-dimensional space. Eqs. (1.1) will be studied with initial condition:

$$\rho(x,0)=\rho_0(x), \quad \rho u(x,0)=\rho_0 u_0(x), \qquad x\in\Omega, \tag{1.3}$$

and one of the following boundary conditions:

$$\begin{cases} u|_{\partial\Omega\times(0,T)}=0, \quad \Omega\subset\mathbb{R}^3 \text{ is a bounded smooth domain;} \\ u|_{\partial\Omega\times(0,T)}=0,\ (\rho,\ u)(x,t)\xrightarrow{|x|\to\infty}(0,0), \quad \Omega\subset\mathbb{R}^3 \text{ is an exterior domain;} \\ (\rho,\ u)(x,t)\xrightarrow{|x|\to\infty}(0,0), \quad \Omega=\mathbb{R}^3. \end{cases} \tag{1.4}$$

In the past decades, there has been a lot of literatures about the mathematical theory of nonhomogeneous incompressible flow. For the case of constant viscosity coefficient, in the absence of vacuum, the global existence of weak solutions as well as local strong solution was established by Kazhikov [1–3]. The uniqueness of the local strong solutions was first proved by Ladyzenskaya–Solonnikov [4] for the initial boundary value problem. When the initial data may contain vacuum states, the global existence of weak solutions was proved by Simon [5]. Under the conditions that the initial data satisfy some compatibility conditions, the local existence of strong solutions was obtained by Choe–Kim [6] (for 3D bounded and unbounded domains) and Lü–Xu–Zhong [7] (for 2D Cauchy problem), respectively. Huang–Wang [8] and Lü–Shi–Zhong [9] obtained the 2D global strong solutions with large initial data for the initial boundary value problem and the Cauchy one, respectively. For the 3D Cauchy problem, Craig–Huang–Wang [10] proved the global well-posedness of strong solutions with suitably small conditions on the initial velocity.

In general, when viscosity $\mu(\rho)$ depends on density ρ, it is more difficult to investigate the well-posedness of system (1.1) due to the strong coupling between viscosity coefficient and density. As for strong solutions away from vacuum, Abidi–Zhang [11,12] proved the global well-posedness for 2D and 3D Cauchy problem under smallness conditions on the initial data. On the other hand, allowing the density to vanish initially, global weak solutions with generally large data were derived by Lions [13]. Later, Desjardins [14] proved the global weak solution with higher regularity for the two-dimensional case provided that the viscosity function $\mu(\rho)$ is a small perturbation of a positive constant in L^∞-norm. Considering the strong solutions with vacuum initially, Cho–Kim [15] obtained the local unique strong solutions to the initial boundary value problem on 3D bounded domain provided that the initial data satisfy some compatibility conditions. Then, Huang-Wang [16] and Zhang [17] prove this local strong solution is indeed a global one under some suitably smallness conditions. However, when $\Omega\subset\mathbb{R}^3$ is an arbitrary smooth domain which can be either bounded or unbounded, the local existence of strong solutions with vacuum and density-dependent viscosity remains open. In fact, this is the main aim of this paper.

Before stating the main results, we first explain the notations and conventions used throughout this paper. For $\Omega\subset\mathbb{R}^3$, $1\le r\le\infty$, and $k\ge 1$, the standard homogeneous and inhomogeneous Sobolev spaces are defined as follows:

$$\begin{cases} L^r=L^r(\Omega), \quad D^{k,r}=D^{k,r}(\Omega)=\{v\in L^1_{\mathrm{loc}}(\Omega)|\nabla^k v\in L^r(\Omega)\}, \\ D^{k,r}_{0,\sigma}(\Omega)=\overline{\{v\in C_0^\infty(\Omega)|\mathrm{div} v=0\}} \text{ closure in the norm of } D^{k,r}, \\ D^k=D^{k,2}, \quad D^k_{0,\sigma}=D^{k,2}_{0,\sigma}(\Omega), \quad W^{k,r}=W^{k,r}(\Omega), \quad H^k=W^{k,2}. \end{cases} \tag{1.5}$$

The definition of strong solutions to Eqs. (1.1) is shown as follows.

Definition 1.1. If all derivatives involved in (1.1) for (ρ,u,P) are regular distributions, and Eqs. (1.1) hold almost everywhere in $\Omega\times(0,T)$, then (ρ,u,P) is called a strong solution to (1.1).

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