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Controlling an alien predator population by regional controls

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ABSTRACT

We investigate the problem of minimizing the total cost of the damages produced by an alien predator population and of the regional control paid to reduce this population. The dynamics of the predators is described by a prey–predator system with either local or nonlocal reaction terms. A sufficient condition for the zero-stabilizability (eradicability) of predators is given in terms of the sign of the principal eigenvalue of an appropriate operator that is not self-adjoint, and a stabilizing feedback control with a very simple structure is indicated. The minimization related to such a feedback control is treated for a closely related minimization problem viewed as a regional control problem. The level set method is a key ingredient. An iterative algorithm to decrease the total cost is obtained and numerical results show the effectiveness of the theoretical results.

A spatially structured SIR problem may be described by the same system; in this case the above mentioned minimization problem is related to the problem of eradication of an epidemic by regional controls.

1. Setting of the problem

Consider the following reaction–diffusion system which describes the dynamics of two interacting populations: prey and predator that are free to move in the habitat $\Omega \subset \mathbb{R}^2$ and are subject to a control acting in a subset $\omega \subset \Omega$ on the predators.

$$\begin{cases} \partial_t h - d_1 \Delta h = r(x)h - \rho(x)h^2 - h(Bp(\cdot,t))(x), & (x,t) \in Q \\ \partial_t p - d_2 \Delta p = -a(x)p + c_0 h(Bp(\cdot,t))(x) + \chi_\omega(x)u, & (x,t) \in Q \\ \partial_\nu h = \partial_\nu p = 0, & (x,t) \in \Sigma \\ h(x,0) = h_0(x), \quad p(x,0) = p_0(x), & x \in \Omega\,. \end{cases} \tag{1}$$

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Here Ω is a bounded domain (open and connected) with a sufficiently smooth boundary $\partial\Omega$, ω is an open subset, $Q = \Omega \times (0, +\infty)$, $\Sigma = \partial\Omega \times (0, +\infty)$. $h(x, t)$ and $p(x, t)$ are the spatial densities at time t of the prey, respectively predator populations. The diffusion coefficients d_1 and d_2 are positive constants, $r(x)$ is the growth rate and $\rho(x)h(x, t)^2$ is a local logistic term of preys. $a(x)$ is the decreasing rate of the predator population. The quantity $h(x, t)(Bp(\cdot, t))(x)$ gives the density of captured prey population at position x, which is transformed into biomass via a conversion rate $c_0 \in (0, +\infty)$. Possible choices of the operator $B \in L(L^2(\Omega))$ will be discussed later.

The homogeneous Neumann boundary conditions describe the no flux of populations across the boundary of the habitat. $h_0(x)$ and $p_0(x)$ denote the initial prey and predator population densities at position x, respectively. The control $u(x, t)$ acts on the predators only, in the spatial subregion ω; χ_ω is the characteristic function of ω.

We assume that

(A1) $r, \rho, a, h_0, p_0 \in L^\infty(\Omega)$, $r(x) \geq r_0,\ \rho(x) \geq \rho_0$ a.e. $x \in \Omega$ (r_0, ρ_0 are positive constants);

$$h_0(x) \geq 0, \quad p_0(x) \geq 0 \qquad \text{a.e. } x \in \Omega,$$

and h_0 and p_0 are not identically zero.

(A2) $B \in L(L^2(\Omega)) \cap L(L^\infty(\Omega))$, $(By)(x) \geq 0$ a.e. $x \in \Omega$, for any $y \in L^2(\Omega)$ such that $y(x) \geq 0$ a.e. $x \in \Omega$.

Two cases are of particular interest to us:

CASE 1. If $(By)(x) = c(x)y(x)$ for $y \in L^2(\Omega)$, where $c \in L^\infty(\Omega),\ c(x) \geq 0$ a.e. $x \in \Omega$, then the functional response to predation is of the usual Lotka–Volterra type.

CASE 2. If $(By)(x) = \int_\Omega \kappa(x, x')y(x')\,dx'$ for $y \in L^2(\Omega)$, where $\kappa \in L^\infty(\Omega \times \Omega)$, $\kappa(x, x') \geq 0$ a.e. $(x, x') \in \Omega \times \Omega$, then the functional response to predation is such that predators, however coming from any position x', upon predation at position x will stay and produce offsprings at this new position (the predators follow the prey). For other prey–predator systems with nonlocal terms see [1].

We may notice that system (1) may also model a spatially structured SIR epidemic system, in which cases $h(x, t)$ and $p(x, t)$ represent the spatial density of the susceptible and infective population, respectively. With $c_0 = 1$, CASE 1 presented above corresponds to a local infection rate, while CASE 2 corresponds to a nonlocal infection rate as proposed by D.G. Kendall [2] (see also [3], and [4]).

For the SIR system $r(x) = b(x) - \mu(x)$, where $b(x)$ is the birth rate and $\mu(x)$ is the natural death rate at position x; $a(x) = \mu(x) + \tilde{\mu}(x)$, where $\tilde{\mu}(x)$ is the additional removal rate due to the extra death rate caused by the disease and the possible natural recovery rate. The infective population do not have offsprings and the recovered individuals acquire immunity. The control u may describe the additional removal of infectives (because of either recovery by treatment or isolation) due to a planned regional intervention by the relevant public health system.

If we view p as an alien pest population density or as an infective population density, then it is of great interest to know if there exists a control u such that, for the solution (h^u, p^u) to (1), $\lim_{t\to\infty} p^u(\cdot, t) = 0$ in an appropriate functional space.

Definition. The predator population is *zero-stabilizable* (eradicable), if for any h_0, p_0 satisfying (A1), there exists $u \in L^\infty_{loc}(\overline{\omega} \times [0, +\infty))$ such that

$$h^u(x, t),\ p^u(x, t) \geq 0 \quad \text{a.e. } (x, t) \in Q\,, \tag{2}$$

and

$$\lim_{t\to+\infty} p^u(\cdot, t) = 0 \quad \text{in } L^\infty(\Omega). \tag{3}$$

We are dealing with zero-stabilizability (eradicability) with state constraints (the states h^u and p^u are subject to the constraints (2)).